

AMERICAN JOURNAL *of* PHYSICS

Devoted to the Instructional and Cultural Aspects of Physical Science

VOLUME 22

NUMBER 2

FEBRUARY 1954



Published nine times per year for the
AMERICAN ASSOCIATION OF PHYSICS TEACHERS
by the

AMERICAN INSTITUTE OF PHYSICS

Incorporated

PRINCE & LEMON STS., LANCASTER, PA., or 57 EAST 55 STREET, NEW YORK 22, N. Y.

BOSTON UNIVERSITY
COLLEGE OF LIBERAL ARTS
LIBRARY

AMERICAN ASSOCIATION OF PHYSICS TEACHERS

OFFICERS

PAUL E. KLOPSTEG, *President*
National Science Foundation
Washington, D. C.

MARSH W. WHITE, *President-Elect*
Pennsylvania State College
State College, Pennsylvania

R. F. PATON, *Secretary*
University of Illinois
Urbana, Illinois

FRANCIS W. SEARS, *Treasurer*
Massachusetts Institute of
Technology
Cambridge, Massachusetts

EXECUTIVE COMMITTEE

V. E. EATON
W. C. MICHELS
WILL V. NORRIS
W. S. WEBB
M. W. ZEMANSKY

The officers and editor are also members of the executive committee.

COUNCIL

K. E. DAVIS
W. GEER
S. O. GRIMM
C. A. HODGES
G. E. C. KAUFFMAN
W. J. MICHELS
W. NOLL
R. R. PALMER
R. L. PRICE
O. L. RAILSBACK
F. VERBRUGGE
I. WALTERSTEIN
L. R. WEBER
C. WILLIAMSON

The members of the executive committee are also members of the council.

AMERICAN JOURNAL OF PHYSICS

Editor

THOMAS H. OSGOOD, *Michigan State College*

Assistant Editor

B. H. DICKINSON, *Michigan State College*

Associate Editors

DAVID L. FALKOFF, *Brandeis University and Massachusetts Institute of Technology*

IRA M. FREEMAN, *Rutgers University*

A. C. HELMHOLZ, *University of California, Berkeley*

W. W. MCCORMICK, *University of Michigan*

NORA M. MOHLER, *Smith College*

C. W. UFFORD, *University of Pennsylvania*

G. H. VINEYARD, *University of Missouri*

C. N. WALL, *University of Minnesota*

ROBERT W. YOUNG, *U. S. Navy Electronics Laboratory, San Diego*

THE AMERICAN JOURNAL OF PHYSICS is devoted to the instructional and cultural aspects of physical science. Previous to 1940 this publication was known as *The American Physics Teacher*. It is published monthly, except during June, July and August, by the American Institute of Physics for the American Association of Physics Teachers at Prince and Lemon Streets, Lancaster, Pennsylvania.

Manuscripts for publication should be submitted to Dr. Thomas H. Osgood, *American Journal of Physics*, Michigan State College, East Lansing, Michigan. The authors' institutions are requested to pay an optional publication charge of \$4 per page which, when honored, entitles them to 100 free reprints. Instructions will be sent with galley proofs.

Proof and correspondence concerning papers in the process of being printed should be addressed to the American Institute of Physics, 57 East 55 Street, New York 22, New York.

Subscription Price

United States and Canada \$6.00
Elsewhere 7.00

Back Numbers

Complete set: Vol. 1, 1933-Vol. 21, 1953—\$147.00

Yearly back number rate: \$8.00

Single copies: \$2.00 each 1933-1936; \$1.50 each thereafter.

Subscriptions, renewals, and orders for back numbers should be addressed to the American Institute of Physics, 57 East 55 Street, New York 22, New York. Members and junior members of the American Association of Physics Teachers receive the *American Journal of Physics*.

Changes of address should be addressed to the American Institute of Physics.

The contents of the *American Journal of Physics* can be found indexed in the *Education Index*.

AMERICAN JOURNAL of PHYSICS

A Journal Devoted to the Instructional and Cultural Aspects of Physical Science

VOLUME 22, NUMBER 2

FEBRUARY, 1954

Physics of the Glassy State.* I. Constitution and Structure

E. U. CONDON

Corning Glass Works, Corning, New York

(Received July 14, 1953)

Modern views of the constitution and structure of inorganic glasses are outlined. Materials used in glass making are classified according to their role as network formers and network modifiers. Evidence of structure from x-ray scattering is reviewed. Examples of linear and nonlinear dependence of specific volume on composition are presented, and the classical factors for composition dependence of heat capacity are related to quantum theory of specific heat.

1. GLASSY STATE

THE glassy or vitreous state of matter is that which arises when a liquid is cooled sufficiently rapidly that it solidifies (becomes rigid) without crystallizing. It is an intermediate state between that of the usual solid and the liquid, possessing the rigidity characteristic of solids and the amorphous disordered structure characteristic of liquids.

Favorable to the production of the glassy state is for the liquid to be relatively viscous at temperatures near the *liquidus*, the temperature at which a solid crystalline phase is in equilibrium with the liquid. The crystalline or devitrified condition is the state of thermodynamic equilibrium so all ordinary glass exists in the vitreous condition because the rates of crystallization have become negligible. However these rates become quite appreciable for glass in the molten condition and are one of the important limiting factors as to the time available for glass forming operations.

Strictly speaking, the viscosity as ordinarily defined—shear stress needed to maintain unit gradient of shear flow—is probably not the rate determining property for the more microscopic flows required for crystal nucleation and growth. Nevertheless there is a close qualitative correlation with ordinary viscosity. The factors governing the kinetics of the devitrification process have not been studied much thus far.

Possibly all liquids could be brought into the glassy state if cooled rapidly enough. Pryde and Jones¹ have reported some observations on water in the vitreous state. Much more readily brought into the glassy state is the more viscous glycerol ($\text{CH}_2\text{OH} \cdot \text{CHOH} \cdot \text{CH}_2\text{OH}$) whose properties have recently been studied by Davies and Jones² and shown to have transformation-range properties like the silica-rich glasses, as discussed in the second lecture.

2. GLASS FORMING OXIDES

Nearly all glasses made industrially are based on silica SiO_2 , the material which exists in the

* Based on the first of four lectures delivered at the Fifteenth Annual Colloquium of College Physicists at the State University of Iowa, June 19 and 20, 1953.

¹ J. A. Pryde and G. O. Jones, *Nature* **170**, 685 (1952).

² R. O. Davies and G. O. Jones, *Proc. Roy. Soc. (London)* **A217**, 26 (1953).

crystalline forms known as quartz, tridymite, and cristobalite. Most of our knowledge of the glassy state is derived from study of such glasses so far that reason primary emphasis is placed on them in these lectures.

When one starts the study of the subject he is likely to be overwhelmed by the great variety of compositions of glass described in the literature, which are hard to remember since they usually involve many components. The mind gropes for a way of bringing this situation into some sort of order which is, of course, to be found in connection with the periodic system of the elements and the modern quantum theory of atomic structure.

The glasses which we shall consider will all be the product of fusing together various oxides. The composition is usually reported by giving the weight-fractions of the various oxide constituents. Some glasses may contain about one percent of fluorine which is usually reported as such with the understanding that in the actual glass two atoms of fluorine replace a single atom of oxygen in the oxide composition as reported. Although the compositions are reported in terms of oxides it needs to be clearly understood at the outset that this fact does not imply that the oxides exist in definite molecular form in the glass.

Although a very large number of oxides find some application in glass-making, only a very few of these play an important role in the subject. For this reason it is possible to start with an extremely abbreviated glassmakers periodic table (Table I). The table is arranged in a way which conforms to the modern theory of atomic structure, identifying the various groups by the kinds of angular momenta of the electronic orbits involved in the outer-electronic shells. The oxides of really major importance are shown in black type. We see that there are four

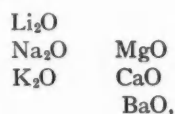
involving incomplete *p* shells, which the chemists call *acidic oxides*:



Of these SiO_2 is most important and P_2O_5 least. Al_2O_3 is listed in parentheses to indicate its amphoteric character in acid-base relationships.

These oxides, especially SiO_2 and B_2O_3 , are known as *network formers*, a term which will become clearer when we consider the modern views of the arrangement of atoms in glasses. Alumina has an intermediate role, partly functioning as a network former.

Other important oxides are known as *network modifiers*. These are:



as well as ZnO and PbO . The other alkalis and alkaline earths are also usable but are not used much in practice because of their comparative rarity. Lead oxide is important in making glasses of high refractive index. The oxides of arsenic and antimony are introduced in small quantities as *fining agents*; they play a role in the oxygen equilibrium of the glass because of the ease with which their valence state can be changed. ZnO is an important constituent of some optical glasses. The oxides of the iron-group metals are important in colored glasses, as are also various materials like cadmium sulphide and cadmium selenide which as colloidal precipitates give rise to the ruby red glasses used to stop auto traffic. Finally Cu, Ag, and Au are important as imparting photosensitive properties to glass as discussed in the fourth lecture.

In spite of the complexity of detail the subject is thus in the main reducible to a consideration of the general properties of three main classes of oxides: the network formers which are principally silica and boric oxide, the network modifiers which are principally of two kinds, the alkali oxides, and the alkaline earth oxides.

3. SOME SPECIFIC GLASS COMPOSITIONS

Fused silica or vitreous silica itself is an excellent glass, but it requires such high tempera-

TABLE I. Glass-makers' periodic table.

	<i>s</i>	<i>d</i>	<i>p</i>	
1	H_2O			
2	Li_2O BeO		B_2O_3	
3	Na_2O MgO		Al_2O_3 SiO_2 P_2O_5	F^-
4	K_2O CaO	(V,Cr,Mn,Fe,Co,Ni)Cu,Zn	GeO_2 As_2O_3 Sb_2O_3	
5	Rb_2O SrO	Ag Cd	SnO_2 Sb_2O_3 Sb_2O_5	
6	Cs_2O BaO	La, Ce	PbO	
7		UO_2		

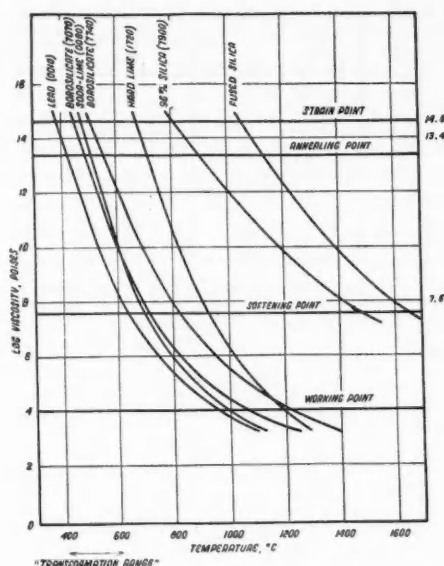


FIG. 1. Viscosity-temperature curves for several kinds of glass.

tures for fusion that its practical usefulness is for this reason severely limited. First and foremost the importance of the modifiers is then as *fluxes*, that is, to reduce the temperature needed to make the glass fluid enough for the various forming operations.

We are not dealing here with sharp melting points. The viscosity varies over many powers of ten in the different conditions of interest. It is customary to describe the viscosity of the glass by giving the temperatures at which the viscosity assumes various arbitrarily chosen standard values. This is illustrated in Fig. 1 which will serve to illustrate the terms and presents actual data on several glasses of considerable importance.

Starting at the highest temperatures, the viscosity of molten glass in tanks is of the order of 10^2 poises, which may be compared with glycerol at 0°C which is 120 poises. When being pressed into molds or drawn into tubes, it is used in the range of temperatures at which its viscosity is 10^3 to 10^6 poises, so arbitrarily the temperature at which $\eta = 10^4$ poises, is called the *working point*. The *softening point* is defined as the temperature at which $\eta = 10^{7.6}$ which corresponds to a condition in which a moderate-sized

piece deforms under its own weight at an easily noticeable rate. The *annealing point* is defined as the temperature at which $\eta = 10^{13.4}$, for this is the temperature at which internal strains are quickly relieved. The *strain point* is the temperature for which $\eta = 10^{14.6}$ and is the highest temperature from which the glass can be rapidly cooled without developing permanent internal strain. The fractional exponents occurring in these definitions came about because the reference temperatures were at first defined in terms of certain practical laboratory test procedures which only later were calibrated in terms of absolute values of viscosity.

The approximate compositions of the glasses shown in Fig. 1 expressed as weight fractions of the constituent oxides are shown in Table II. In Fig. 1, fused silica and 7900 stand out as being very much harder than the others. It is interesting to note nevertheless how much softer the 7900 glass is than fused silica.

1720 is a hard-lime glass of high alumina content of a type used for cooking utensils.

7740 is a borosilicate glass that is widely used for all forms of chemical glassware.

7070 is a borosilicate glass of high electrical resistance specified for certain electrical applications.

0080 is typical of a soda-lime glass as used for electric lamp bulbs, window glass, and bottles. Glasses close to this in composition account for 90 percent of all glass melted.

0010 is a potash-soda-lead glass of high electrical resistance as used for electric lamp bulb stems and sign tubing.

In the first approximation the softness of the glass is mainly dependent upon the SiO_2 content

TABLE II. Approximate compositions of the glasses of Fig. 1.

Glass	Network formers			Alkalies		Alkaline earths		Lead
	SiO_2	B_2O_3	(Al_2O_3)	Na_2O	K_2O	MgO	CaO	
Fused silica	0.998 +					(0.002)		
"Vycor" 7900	0.96 +	0.03 +	(0.01 -)					
1720	0.57	0.04	0.20	0.01		0.12	0.06	
"Pyrex" 7740	0.80	0.14	0.02	0.036	0.004			
0080 Soda-lime	0.72		0.01	0.20		0.03	0.04	
7070	0.71	0.27		(0.02)				
0010	0.63		0.01	0.03	0.06		0.01	0.21
Lead								

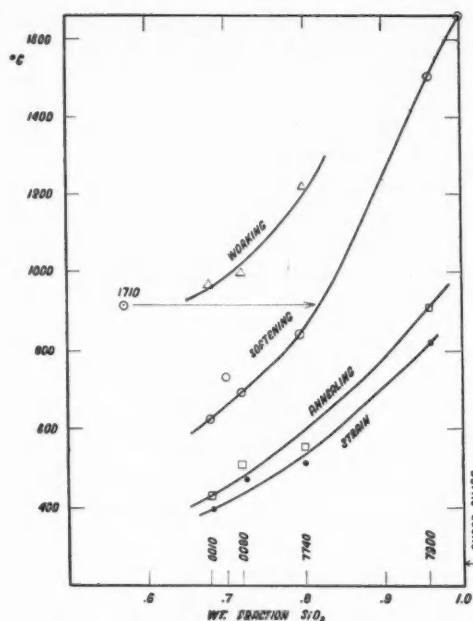


FIG. 2. Dependence of hardness on silica content.

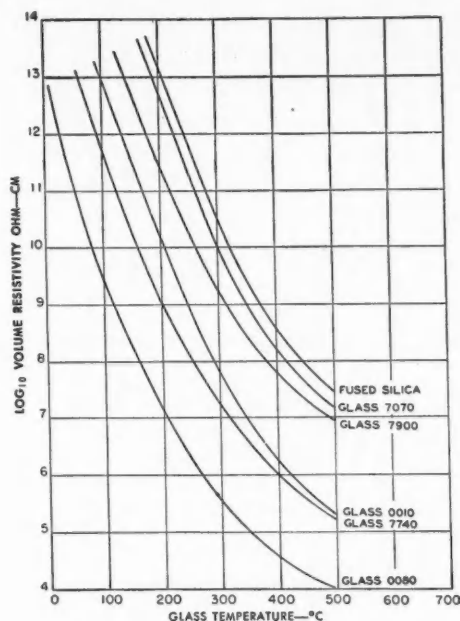


FIG. 3. Volume resistivity vs temperature.

expressed as weight percent as is indicated in Fig. 2, in which the various characteristic temperatures for these glasses are plotted against the weight fraction of SiO_2 .

The differences in the electrical volume resistivity of these glasses are brought out in Fig. 3 which shows the log resistivity against temperature for the same glasses.

From the foregoing it is evident that the properties are strongly dependent on the composition which therefore has to be carefully controlled to produce product with properties within narrowly specified limits. The strong dependence on composition makes possible the development of glasses having widely different properties adapted to special uses.

4. THE GLASS NETWORK

We turn now to the modern ideas of the network constitution of glass.³ These are an out-

³ The following review papers are recommended: H. G. Vogt, *Encyclopedia of Chemical Technology* (Interscience Publishers, Inc., New York), Vol. 7, pp. 195-206; B. E. Warren, *J. Am. Ceram. Soc.* **24**, 256 (1941); B. E. Warren, *J. Appl. Phys.* **13**, 602 (1942); K. H. Sun, *J. Am. Ceram. Soc.* **30**, 277 (1947); as well as the following book references: W. A. Weyl, *Coloured Glasses* (Society of Glass Technology, Sheffield, 1951), Chap. 2; J. E. Stanworth, *Physical Properties of Glass* (Clarendon Press, Oxford, 1950), Chap. 2; J. M.

growth of the study of silicate crystalline materials by means of x-ray diffraction analysis, the general ideas on the subject laid down in a classic paper by Zachariasen,⁴ and their fuller working out in a series of experimental researches by Warren and his associates⁵ which were a natural outgrowth of the pioneering researches on x-ray diffraction in liquids made at the State University of Iowa by Stewart⁶ and his associates about a quarter century ago.

Let us first recall briefly what one can learn from x-ray diffraction studies. Each electron in the material scatters⁷ coherently with a cross

Stevens, *Progress in the Theory of the Physical Properties of Glass*, (Elsevier Press, New York, 1948), Chap. 1.

⁴ W. H. Zachariasen, *J. Am. Chem. Soc.* **54**, 3841 (1932).

⁵ B. E. Warren, *J. Appl. Phys.* **8**, 645 (1937) (Vitroous Silica); B. E. Warren and C. F. Hill, *Z. Krist.* **89**, 481 (1934) (GeO_2 Glass); B. E. Warren, H. Kruther and O. Morningstar, *J. Am. Ceram. Soc.* **21**, 259 (1938) (Soda-silica Glass); B. E. Warren and J. Biscoe, *J. Am. Ceram. Soc.*, **21**, 287 (1938) (Soda boric oxide Glass); Biscoe, Pincus, Smith, and Warren, *J. Am. Ceram. Soc.* **24**, 116 (1941) (Phosphate Glasses); J. Biscoe, *J. Am. Ceram. Soc.* **21**, 262 (1941) (Soda-lime-silica Glasses); R. L. Green, *J. Am. Ceram. Soc.* **25**, 83 (1942) (Potash-boric oxide Glass); G. J. Bair, *J. Am. Ceram. Soc.* **19**, 339 (1936) (PbO-silica Glasses).

⁶ G. W. Stewart, *Revs. Modern Phys.* **2**, 116 (1930); *Phys. Rev.* **29**, 232 (1927); **31**, 174 (1928); **35**, 726 (1930); **37**, 9 (1931), and others.

⁷ Compare A. H. Compton and S. K. Allison, *X-Rays in Theory and Experiment* (D. Van Nostrand Company, Inc., New York, 1935), Chap. III.

section $\sigma = (e^2/mc^2)^2$ according to the theory of J. J. Thomson, and the total observed scattering intensity in any direction is obtained by the addition of *amplitudes* of scattered waves—it is this fact which makes the resultant be dependent on the structure in the scatterer. Let the wave vector for the incident wave be \mathbf{k} and for the scattered wave be \mathbf{k}' , where both have the magnitude $2\pi/\lambda$, and suppose that $\rho(\mathbf{r})$ is the volume density of electrons at the vector position \mathbf{r} . Then the *amplitude* of the scattered wave due to coherent scattering by the electrons will be proportional to

$$\iiint \rho(\mathbf{r}) e^{i(\mathbf{k}-\mathbf{k}') \cdot \mathbf{r}} dV,$$

the volume integral extending over the whole of the scatterer that is illuminated by the incident beam.

Thus the amplitude of the scattered wave is proportional to the $(\mathbf{k}-\mathbf{k}')$ component of the Fourier transform of $\rho(\mathbf{r})$. For scattering through the angle 2θ (Fig. 4), this is equivalent to reflection from the planes normal to $(\mathbf{k}-\mathbf{k}')$ in accordance with the well-known Bragg law. Since $|\mathbf{k}-\mathbf{k}'| = (4\pi \sin\theta)/\lambda$ the scattering through angle 2θ arises from the component of the Fourier transform of the electron density which has a wavelength in the material equal to $\lambda/2 \sin\theta$. In other words, small-angle scattering is related to the part of the structure involving periodicities over larger distances.

Looked at in this way there is an essential unity between scattering by amorphous liquids, by polycrystalline solids, and by single crystals. In the latter case the material is quite accurately periodic in its structure so that scattering is almost completely absent except in the allowed directions for which $(\mathbf{k}-\mathbf{k}')$ in magnitude and direction agrees with one of the actual perio-

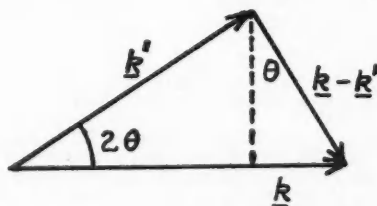


FIG. 4. Scattering of x-rays by electrons.

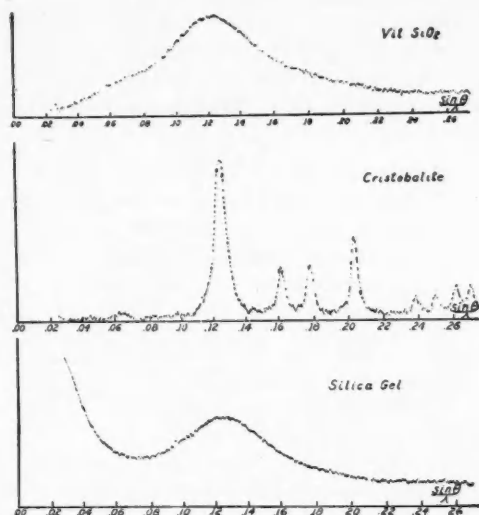


FIG. 5. Microphotometer records of x-ray diffraction patterns of vitreous silica, cristobalite, and dried commercial silica gel.

dicities in the lattice. In the powder method of Debye and Scherrer, which averages over many randomly-oriented tiny crystals, the conditions are such that the condition on $(\mathbf{k}-\mathbf{k}')$ as to direction is always fulfilled by the fact that some of the scatterer is properly oriented, but the condition on $|\mathbf{k}-\mathbf{k}'|$ is determined by the definite lattice spacing of the tiny crystals thus giving rise to sharp rings. In amorphous materials the scattering density function $\rho(\mathbf{r})$, being aperiodic, is represented by a Fourier integral rather than a Fourier series so that scattering occurs for a continuum of values of $|\mathbf{k}-\mathbf{k}'|$ giving rise to broad, fuzzy rings.

By measuring the scattered intensity at various angles and performing a Fourier transform one can find the distribution of scattering power in the amorphous material. Figure 5 shows the microphotometer record of the intensity of scattering as a function of $\sin\theta/\lambda$ as measured by Warren for vitreous silica, cristobalite, and silica gel. It is remarkable that the broad peak in the two amorphous forms came at nearly the same value of $\sin\theta/\lambda$ as the main sharp peak in the crystalline cristobalite. Also the presence of a large amount of small angle scattering in silica gel in the range for $\sin\theta/\lambda$ from 0.02 to 0.06 points to the existence of structure on a scale of

periodicity from 8 to 25Å that is absent in the vitreous silica and in the crystal.

From more detailed analysis of the vitreous silica curve along the lines indicated, it is inferred that

- the average Si—O distance is 1.62Å in close agreement with 1.60Å found in crystalline silicates, and
- the average number of oxygens surrounding each Si is 4, closely resembling crystalline silicates.

These two facts give rise to a picture of vitreous silica as being built up of a random distribution of tetrahedral (SiO_4) groups interlocking in the sense that each O belongs to two adjacent tetrahedra. The tetrahedra are thus linked together to form one giant molecule, a fact which correlates with the strength and hardness of silica in all its forms.

It is customary to discuss the structure as if the individual atoms are ions held together by ionic bonds. On this view (SiO_4) is an ionic tetrahedral structure with a central Si^{++++} surrounded by four O^{--} ions each of which obtained one of its electrons from each of the two silicons to which it is bonded. Throughout inorganic crystal chemistry the ions tend to keep rather constant ionic radii, tables of which have been made by Pauling,⁸ based on an earlier tabulation by Goldschmidt, which is exhibited graphically in periodic table form for the ions of interest in Fig. 6. Of interest here is that B^{3+} , Si^{4+} , and Al^{3+} are all small compared with O^{--} , in contrast with the larger cations of lower charge number

which enter into the modifier oxides. In Fig. 7 a plan view of a number of interlocking SiO_4 tetrahedral units is drawn. We regard vitreous silica then as an extended random network of such tetrahedra joined together by shared oxygen atoms or ions.

For brevity, we use the ionic radii and the terminology of ionic bonds as is customary in the literature of glass. The more modern viewpoint regards the bonds as partly ionic and partly covalent in character.⁹ Thus if ψ_1 represents a wave function for the ionic structure of SiO_4 and ψ_2 that for a covalent bond structure, a better approximation is an appropriate linear combination $\psi = (a\psi_1 + b\psi_2)$ determined by resonance energy calculations. Here $|a|^2$ is regarded as the fractional ionic character. Pauling has attempted to estimate the fractional ionic character for various links with the following results of interest in glass-making shown in Fig. 8. According to this, the basic SiO bond is only 50 percent ionic, so that the usual ionic descriptions are not to be taken literally.

Corresponding work on the $\text{Na}_2\text{O}-\text{SiO}_2$ glasses, up to 0.35 Na_2O (wt.) showed the same SiO_4 tetrahedral structure in the main. However, as there are now more oxygens than in SiO_2 , it is not possible for all to be bonded between two Si as in silica; some must be bonded to only one Si, and it is to this fact that the softening action of Na_2O is due. The diffraction work gives further indication of 2.35Å for the average Na—O distance as would be expected from the ionic radii of Fig. 6 and also gives an average coordination of 6 ± 1 oxygens around each sodium atom.

The soda-boric acid series introduces a new feature. Diffraction results indicate¹⁰ that vitreous boric oxide itself is made up of a random network of BO_3 triangles linked together by shared oxygens, analogously to the linkage of the SiO_4 tetrahedra, the B—O distance being 1.39Å, rather less than the 1.60Å expected from

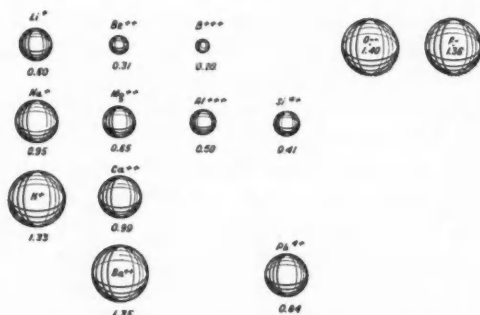
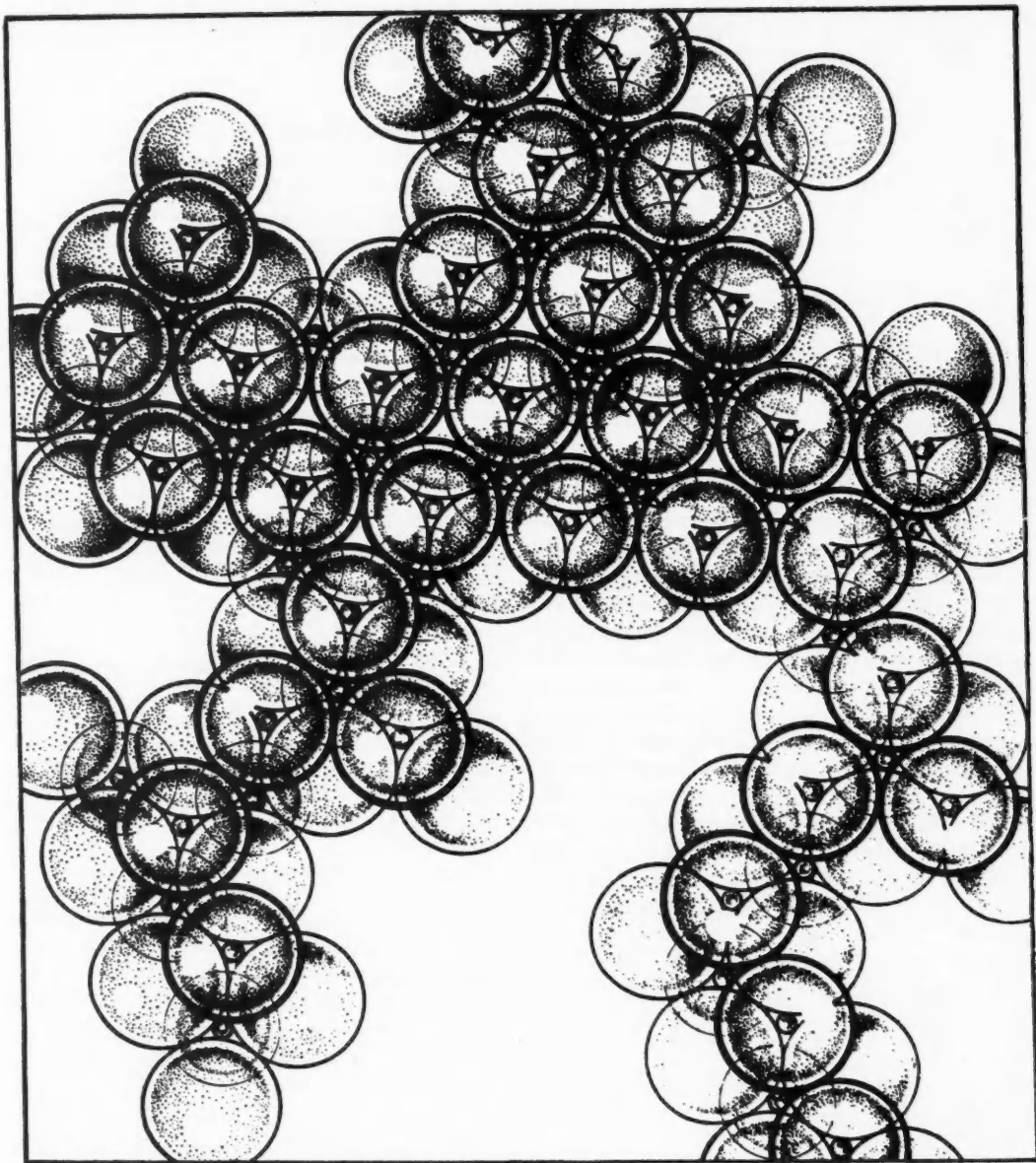


FIG. 6. Ions of importance in glass structures.

⁸ L. Pauling, J. Am. Chem. Soc. 49, 765 (1927).

⁹ For an excellent up-to-date survey see the book by C. A. Coulson, *Valence* (Oxford University Press, New York, 1952).

¹⁰ See, however, K. Fajans and S. W. Barber, J. Am. Chem. Soc. 74, 2761 (1952), who conclude that B_2O_3 is made up of molecular dimers B_2O_6 relatively loosely held together, accounting for its low melting point relative to that of fused silica.

FIG. 7. Groups of SiO_4 units.

the Pauling ionic radii. A typical such triangular unit is shown in Fig. 9.

With the addition of Na_2O , it is found that the B—O distance gradually increases up to 1.48Å and the coordination number definitely increases

from 3 to 4 indicating a *gradual shift in the structure* from one of interlocking BO_3 triangles over to one of partially interlocking BO_4 tetrahedra. A similar behavior was observed in an x-ray diffraction analysis of several glasses in the

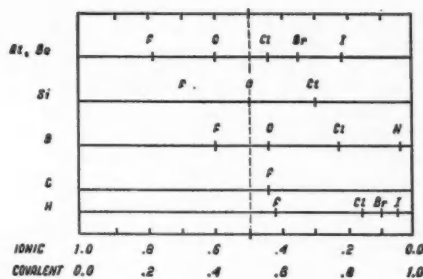


FIG. 8. Ionic-covalent character of chemical bonds.

$K_2O-B_2O_3$ series. This behavior is the basis for many peculiar properties of borosilicate glasses, known in the literature broadly as the *boric oxide anomaly*.

5. DENSITY OF GLASS

The older glass literature abounds with papers which attempt to express various properties of glass as linear functions of the composition. These attempts are useful for concise presentation of empirical data and also in serving to direct attention to unusual cases involving structural peculiarities. Let us consider the density of glass from this point of view.

In physical chemistry we learn that the molal volume of many compounds and solutions is a sum of contributions from the constituents. Hence if x_r is the weight fraction of the r th oxide in a glass whose specific volume is v ($=1/\rho$, where

ρ is the density), it is natural to suppose that, in a first approximation,

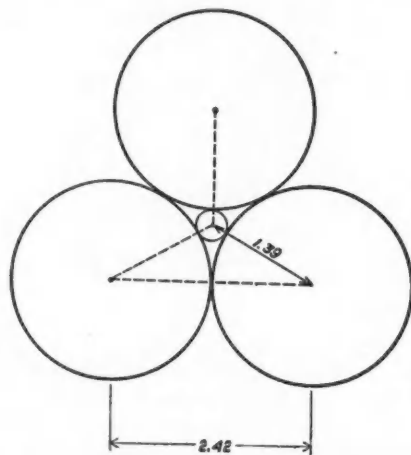
$$v = \sum_r v_r x_r,$$

where v_r is the effective specific volume of the r th oxide in glass. This kind of formula is discussed in Morey,¹¹ Chap. X.

We have seen that the largest ion present in glass is the oxygen atom. The network forming cations are much smaller. It is therefore interesting to consider the density data on a uniform basis with regard to the volume contributed by one gram molecular weight of oxygen. For an oxide whose formula is written A_aO_b let M_r be $1/b$ times the usual molecular weight, so M_r grams of the r th oxide contain 1 gram mole of oxygen atoms. If v_r is the specific volume of the

TABLE III. Volume (cm^3) per gram mole of oxygen atoms.

Oxide	Free oxide	Effective glass factors			
		Winkelmänn and Schott	Baillie	English and Turner	Tillotson
SiO_2	13.65 (vitr.)	13.05	13.40	13.65	13.05
B_2O_3	12.60	12.25			
Al_2O_3	8.53	8.34	12.40	12.40	12.40
P_2O_5	11.90	11.15			
Li_2O	14.9				8.06
Na_2O	27.3	23.9	19.4	17.9	
K_2O	40.6	33.6			
MgO	10.9	10.6	12.4	11.9	10.1
CaO	16.5	17.	13.05	11.2	13.7
BaO	26.8	21.9			
ZnO	14.85	13.8			
PbO	23.4	23.2			

FIG. 9. BO_3 triangular unit.

free oxide, then $v_r M_r$ is the volume in cm^3 of an amount containing one mole of oxygen atoms. Alternatively, if v_r is the effective specific volume of the oxide in a glass, $v_r M_r$ is the volume per mole of oxygen for that oxide constituent. Various sets of effective factors are presented in Morey's Table X.1 from the literature. These were used to prepare the values of Table III.

These factors are the values to be used in the linear formula expressed as

$$v = \sum_r (v_r M_r) (x_r / M_r),$$

where (x_r / M_r) is the number of moles of oxygen in unit weight of glass coming from the r th oxide.

¹¹ G. W. Morey, *The Properties of Glass* (Reinhold Publishing Corporation, New York, 1938), p. 224.

Aside from the fact that the different authors give rather different values, we note that

- the effective values for network formers are close to the value for the free oxide, and
- for the alkalis the effective value is decidedly less than the free oxide value.

Both these observations are qualitatively consistent with the view that the alkali ions fill in the interstices in the network and so do not add much to the space occupied by the glass. Note how much smaller is the value for the network formers than it is for water (18 cm^3), serving to emphasize the known fact of the open structure of liquid water compared even with the open structure of the glass networks.

In recent years a good deal of attention¹² has been given to departures from the linear formula. We shall merely consider three binary systems and one ternary system to provide simple examples of how it applies and how it breaks down. The first example is of the binary glasses made of two network formers SiO_2 and B_2O_3 , illustrated in Fig. 10. Ordinates are the volume occupied by enough glass to contain one mole of oxygen atoms and the scale of abscissas shows the fraction u of all the oxygen atoms contributed by the silica

$$u = (x_1/M_1)/[(x_1/M_1) + (x_2/M_2)],$$

$$V = (1/\rho)/[(x_1/M_1) + (x_2/M_2)],$$

where the subscript 1 refers to silica and 2 to boric oxide. The experimental points lie quite nicely on the straight line joining the values derived from the pure vitreous oxides at either end. [The experimental points are calculated from a series given in Morey,¹¹ (p. 231).]

The second example provides a striking contrast. It is the series SiO_2 and Na_2O consisting of a network former with increasing amounts of network modifier. The points in Fig. 11 are calculated in the same way as before from a series given in Morey,¹¹ (p. 232), where now subscript 1 refers to soda and 2 to silica. The line with an arrowhead is the one joining to the volume 27.3 cm^3 of pure Na_2O . The experimental points in the range $0 < u < 0.14$ lie on a straight

¹² J. M. Stevels, see reference 3, Chap. 2. M. L. Huggins and K. H. Sun, *J. Am. Ceram. Soc.* 26, 4 (1943); 27, 10 (1944).

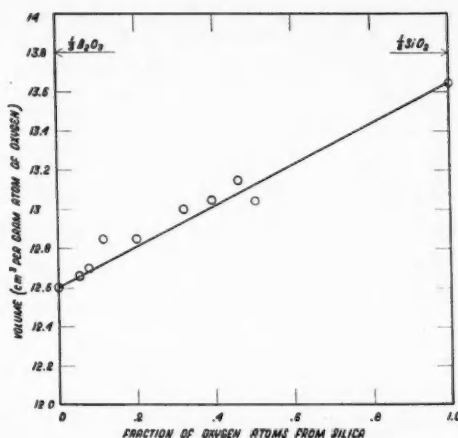


FIG. 10. Molar volumes in the silica-boric oxide system.

line of much lower slope, appropriate to an effective molal volume of only 17.65 cm^3 for Na_2O dissolved in the SiO_2 vitreous network. For $u > 0.14$ the curve is again straight but with a considerably greater slope indicating a swelling of the network after about one in seven oxygens are contributed by the soda—otherwise expressed, the abrupt change comes about where there are two sodium atoms for every three (SiO_4) tetrahedra, so one would expect most of the interstices to be filled.

The second example exemplifies well (a) the departure of the effective volumes from the free oxide values and (b) the nonlinearity of behavior of a system containing a network modifier in a network former.

The third example is the binary system ($\text{B}_2\text{O}_3 + \text{Na}_2\text{O}$), calculated from the new data

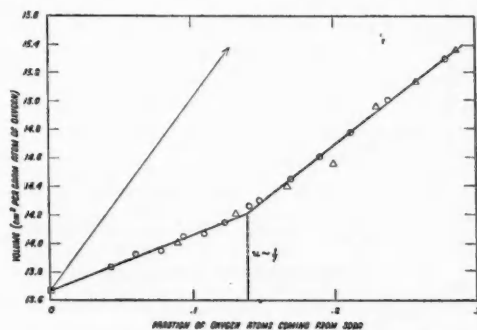


FIG. 11. Molar volumes in the silica-soda system.

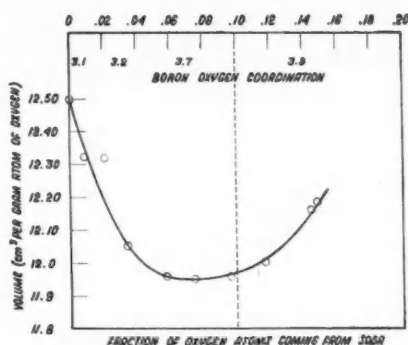
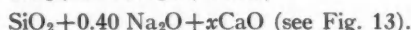


FIG. 12. Molar volumes in the boric oxide-soda system.

of Shartsis, Capps, and Spinner.¹³ In Fig. 12, V is plotted against u , as before.

Addition of Na_2O causes the total volume per oxygen atom to contract up to about $u \sim 0.07$ at least, the total shrinkage being some 5.5 percent of the initial value. The minimum volume occurs for compositions near to $\text{Na}_2\text{O} \cdot 4\text{B}_2\text{O}_3$ and $\text{Na}_2\text{O} \cdot 5\text{B}_2\text{O}_3$.

The fourth example is based on data by Peddle (see Morey,¹¹ p. 376) for the two series (expressed in moles):



The fraction of all oxygen atoms coming from the lime (in the first series) is

$$u = x / (2.2 + x),$$

and the associated volume per mole of oxygen atoms is

$$V = (72.4 + 56.1x) / (2.2 + x),$$

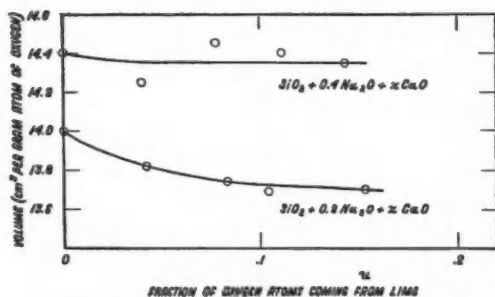


FIG. 13. Molar volumes in some soda-silica glasses.

¹³ Shartsis, Capps, and Spinner, *J. Am. Ceram. Soc.* **36**, 35 (1953).

and similarly for the second series. Incidentally, on p. 376 is also given¹¹ another set of data for $(\text{SiO}_2 + x\text{Na}_2\text{O})$ which checks perfectly the other series in Fig. 11, the points from this series being plotted as triangles with center dots on Fig. 11. Here there is definite contraction in the first series with increasing proportion of lime and an almost exact constancy in the second series. This is associated with the effect of the double valence of Ca so it can act to pull the network together, and the fact that there is only one such ion to be accommodated for each lime-oxygen.

In conclusion, it may be said that the traditional linear specific volume factors are too inaccurate to be very useful, and they fail to draw attention to important facts related to changes in the glass network with changes in composition.

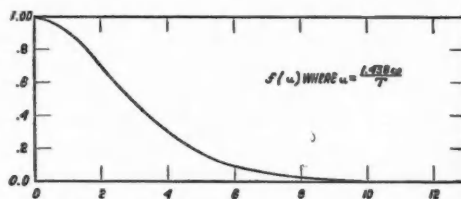


FIG. 14. Quantum specific heat function.

6. HEAT CAPACITY OF GLASS

Data on heat capacity of glass are understandable in terms of the network structural ideas. According to Morey¹¹ (Chap. VIII), the specific heat may be more accurately represented as a linear function of the composition than most other properties.

According to the quantum theory of heat capacity, we expect that at sufficiently high temperatures, but such that the glass is still quite rigid, each atom will behave like a three-dimensional harmonic oscillator. Classically such an oscillator has a heat capacity of $3R$ or $6 \text{ cal}/(\text{mole deg})$ in the limit of high temperatures. According to quantum theory the actual specific heat will be less than this:

$$C = 3Rf(u),$$

where

$$f(u) = \frac{u^2 e^{-u}}{(1 - e^{-u})^2}, \quad u = hc\omega/kT,$$

if ω is the frequency of oscillation in cm^{-1} . Also $hc/k=1.438$. In Fig. 14 is a graph of $f(u)$ against u .

In an actual solid different atoms will be differently bound so that the actual specific heat will be given by a sum of functions $f(u_1)+f(u_2)+\dots$, where the u 's correspond to their different frequencies $\omega_1, \omega_2, \dots$. In the modern theory of crystal lattices, much attention has been given to calculations of the distribution of frequencies to be used.

Measurements of the heat capacity of vitreous silica and vitreous boric oxide over quite a range of temperatures are reported by Morey and are plotted in Fig. 15. Here the ordinate is the value of the specific heat, not in cal/g deg, but in the ratio of the actual value to the classical theoretic-

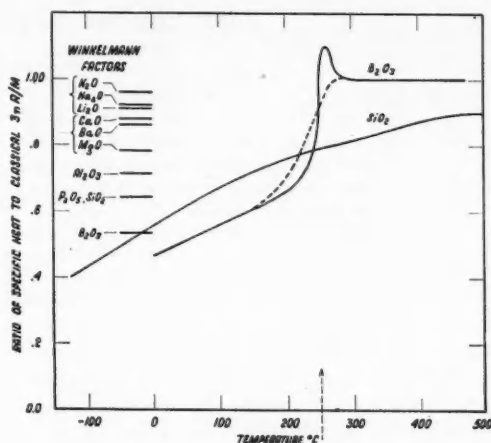


FIG. 15. Heat capacity of SiO_2 and B_2O_3 and specific heat factors.

TABLE IV. Heat capacity factors of oxides in glass.

Oxide	Winkelmänn Observed factor	Ratio to $3nR/M$
SiO_2	0.1913	0.64
B_2O_3	0.2272	0.53
Al_2O_3	0.2074	0.71
P_2O_5	0.1902	0.64
Li_2O	0.5497	0.91
Na_2O	0.2684	0.92
K_2O	0.1860	0.96
MgO	0.2439	0.78
CaO	0.1903	0.88
BaO	0.067	0.86
ZnO	0.1248	0.84
PbO	0.0512	0.95

cal limit, that is, to $3nR/M$, where $n=3$ for SiO_2 and 5 for B_2O_3 and M is molecular weight.

The curve for SiO_2 is smooth and of the type usually shown by crystalline solids and to be expected from the quantum theory of specific heat. On the other hand, that for B_2O_3 , while showing a similar behavior below 200°C , shows a relatively sharp peak at about 260°C and then levels off at the full classical value for temperatures above 300°C . The melting point is 577°C . The exact shape of the curve depends on the previous thermal history of the sample as indicated by the two different representative curves.

These curves indicate that in SiO_2 and B_2O_3 such strong interatomic forces prevail that the

atomic oscillation frequencies are high enough that the heat capacities at room temperature are only about 0.6 and 0.5, respectively, of the classical limit.

It is now of interest to see how the traditional Winkelmänn factors for specific heat compare with the classical values, $3nR/M$, for each oxide. The factors c_r (cal/g deg) for an additive representation of the heat capacity,

$$c = \sum c_r x_r,$$

in terms of weight fractions x_r of the oxides in a glass are taken from Table VIII-1 in Morey¹¹ and put here in Table IV along with the ratio of the factor to the $3nR/M$ classical limit.

These results correlate clearly with what would be expected from the network-structure theory. The strongly bonded atoms in the network are considerably affected by the quantization of the oscillator energy levels, to the extent that their effective heat capacity is some 50 percent to 70 percent of the classical value, the alkaline earths contributed much more nearly the classical value and the alkalis contribute almost completely what would be expected, corresponding to the fact that they are much more loosely bound in the network. The observed ratios are shown on the left side of Fig. 15.

Films Selected for College Physics Courses

ROBERT L. WEBER

The Pennsylvania State College, State College, Pennsylvania

(Received July 21, 1953)

Some 190 instructional films in physics are classified and listed as being well suited for college courses in physics, both at the introductory and advanced levels. Sources for purchase or loan of the films are included.

SINCE the 1949 listing¹ of physics instructional films in this *Journal*, important new films have been made available. Notable among these are films prepared by U. S. Government agencies and those planned by the Visual Aids Committee² of the AAPT. These new materials, and the interest evidenced by continuing requests for reprints of the original list, are the reasons for offering a new list of films suitable for college physics classes.

The films were selected, from some 200 viewed, as reasonably accurate, relatively free of extraneous material, and readily available. Those marked with an asterisk (*) are especially recommended as providing effective summaries of topics included in most *first-year* college physics courses. Films marked with a dagger (†) are outstanding films suited to more *advanced* courses.

The films in this list are 16-mm sound films in black and white, unless otherwise designated. Each title is followed by the name of the producer and the principal distributor from whom the film can be purchased. Addresses are given at the end of the film list. In many cases the film can also be borrowed from local film distributors³ or from film libraries.⁴

Coordinated filmstrips and instructor's manuals are available for all of the U. S. Office of Education films. Instructor's manuals, some including the complete spoken commentary, are obtainable for many of the other films. When

such a booklet is known to be available, the word "Guide" is included in the listing.

The decimal classification numbers are from the 1951 Dewey Standard Decimal Classification. These numbers may serve to specify the content of the film and may aid in locating films in certain directories.

The 3×5-in. film catalog cards prepared by the U. S. Office of Education Visual Education Service may be of interest to users of physics films. The cards include summaries of film content. They provide a neat way of cataloging films owned or frequently borrowed. The film catalog cards can be obtained by writing the Library of Congress, Card Division, Washington 25, D. C.

The selection of films for this list, and the "starring" of some titles, are based on the experience and opinion of one instructor. He would welcome critical comments from other teachers about these or other films, especially with reference to the general criticism and ideals for instructional films expressed in the original listing.¹

Introduction

	Decimal Classification
Introduction to physics. 10 min, b&w or color, (1949). Guide. Coronet.	530
*An introduction to vectors—coplanar, concurrent forces. 22 min, (1945). Guide. USOE, United World Films.	512
The micrometer. 15 min, (1942). Guide. USOE, United World Films.	389
Precise measurements for engineers. 33 min, (1949). Describes work of the National Physical Laboratory, England. Data Films, British Information Services.	389
Precisely so. 20 min, (1940). General Motors.	389
*The slide rule (the "C" and "D" scales, multiplication and division). 24 min, (1944). Guide. USOE, United World Films.	510.81

¹ R. L. Weber, *Am. J. Phys.* **17**, 408–412 (1949).

² Mark W. Zemansky, Chairman; Vernet E. Eaton, Walter C. Michels, Robert Petry, Eric M. Rogers, Robert L. Weber, and Robert H. Randall.

³ Frederic A. Krahn, *Educational Film Guide* (H. W. Wilson Company, 950 University Avenue, New York 52), includes appraisals and directory of distributors.

⁴ Seerley Reid and Anita Carpenter, *A directory of 2660 16 mm Film Libraries*, U. S. Dept. of Health, Education, and Welfare. Purchase copies from U. S. Government Printing Office, Washington 25, D. C.

- The slide rule** (proportion, percentage, squares and square roots). 21 min, (1944). Guide. USOE, United World Films. 510.81
- Time**. 20 min, (1944). Elgin, Modern. 681.1
- Vectors**. 12 min, (1945). USN, MN-1540-n; United World Films. 512

Mechanics

- Aerodynamics: air flow**. 18 min, (1943). Guide. AAF, United World Films. 629.123
- Aerodynamics: forces acting on an air foil**. 27 min, (1941). Guide. AAF, United World Films. 629.123
- Air in action**. 10-min, b&w or color, (1947). Guide. Wind tunnel demonstrations, chiefly for high school physics classes. Coronet. 629.123
- Analytical balance technique**. 30 min, si, (1941). United World Films. 389
- Application of Pascal's law: Parts I, II**. 12, 15 min, (1943). USN, United World Films. 532
- Attitude gyro**. 18 min, (1945). Sperry. 629.15
- Basic hydraulics**. 9 min, color, (1944). Adel for USN, MN-5027-a; United World Films. 532
- Basic principles of the analytical balance**. 19 min, (1951). USPHS, M-12; United World Films. 389
- Derivation of Pascal's law: Parts I, II**. 16, 18 min, (1943). USN, MN-1730-a, b; United World Films. 532
- Diesel—the modern power**. 26 min, (1939). General Motors. 621.436
- Fluid flow in hydraulic systems**. 9 min, color, (1944). Adel for USN, MN-5027-b; United World Films. 532
- Force and motion**. 10 min, b&w or color, (1949). Guide. Coronet. 531
- Gyro Compass**. 10 min, (1943). Sperry. 629.15
- Gyroscope and the earth's rotation**. 10 min, (1944). USN, MN-1792-b; United World Films. 629.15
- Gyroscope and gravitation**. 14 min, (1944). USN, MN-1792-c; United World Films. 629.15
- Mechanics of liquids**. 10 min, b&w or color, (1949). Guide. Coronet. 532
- Nature of energy**. 10 min, b&w or color, (1949). Guide. Coronet. 531
- New frontiers in space**. 26 min, (1953). March of Time. 520
- Principles of dry friction**. 17 min, (1945). Guide. USOE, United World Films. 531
- Principles of lubrication**. 16 min, (1945). Guide. USOE, United World Films. 621.89
- *Principles of moments**. 23 min, (1945). Guide. USOE, United World Films. 531
- Progressive waves: transverse and longitudinal**. 8 min, (1953). AAPT, McGraw-Hill. 530.1
- Romance of the gyroscope**. 11 min, (1943). Sperry. 629.15
- *Simple harmonic motion**. 8 min, (1953). AAPT, McGraw-Hill. 530.1
- Simple machines**. 11 min, (1942). Encyclopaedia Britannica Films. 531
- Stationary longitudinal waves**. 8 min, (1953). AAPT, McGraw-Hill. 530.1
- Stationary transverse waves**. 8 min, (1953). AAPT, McGraw-Hill. 530.1
- Sperry Gyrosyn compass**. 25 min. (1946). Sperry. 629.123
- Tension testing**. 21 min, (1944). Guide. USOE, United World Films. 620.112
- *Uniform circular motion**. 8 min, (1952). AAPT, McGraw-Hill. 530.1

Heat and Thermodynamics

- *Carnot cycle** (Kelvin temperature scale). 8 min, (1952). AAPT, McGraw-Hill. 621.4
- Construction of Diesel engines**. 17 min, (1945), USN, United World Films. 621.436
- Diesel engine** (Ideal Diesel cycle). 8 min, (1952). AAPT, McGraw-Hill. 621.436
- Gasoline engine** (Otto cycle). 8 min, (1952). AAPT, McGraw-Hill. 621.4
- Harnessed lightning**. 25 min, color, (1951). Jet engines. General Motors. 629.14353
- *Heat and its control**. 40 min, (1937). Johns-Manville. 536
- Heat and its control**. 20 min, (1942). A shorter version of the preceding film. USBM. 536
- Heat rays**. 10 min, (1952). Guide. G.B. Equipments, F4735. 536
- Jet propulsion**. 15 min, color, (1946). General Electric. 629.1435
- Molecular theory of matter**. 10 min, (1932). Encyclopaedia Britannica Films. 541.2
- Nature of heat**. 10 min, b&w or color, (1953). Guide. Coronet. 536
- Properties of water**. 11 min, b&w or color, (1941). Guide. Coronet. 543.13
- Ram jet story**. 24 min, color, (1952). Wright Aeronautical. 629.14353
- Steam turbine**. 11 min, (1947). Guide. G.B. Equipments, C3582. 621.1
- Thermodynamics**. 11 min, (1938). Encyclopaedia Britannica Films. 536.4
- Tornado in a box**. 28 min, (1946). The gas turbine. Allis-Chalmers. 621.4
- Turbo-jet propulsion**. 16 min, (1951). Guide. G.B. Equipments, H2994. 629.1435

Meteorology

- Atmosphere and its circulation**. 11 min, (1945). Encyclopaedia Britannica Films. 533
- Clouds**. 11 min, (1939). USDC, Weather Bureau, United World Films. 551.5
- Cold Front**. 19 min, color, (1943). Walt Disney for USN Bureau of Aeronautics. Museum of Modern Art. 551.591

- Fog.** 25 min, color, (1943). Walt Disney for 551.572
USN Bureau of Aeronautics. Museum of
Modern Art.
- †**Modern weather theory: development and** 551.5
characteristics of atmospheric waves. 15
min, (1940). AAF, United World Films.
- †**Modern weather theory: primary circulation.** 551.5
19 min, (1940). AAF, United World Films.
- Story of a storm.** 10 min, b&w or color, 551.5
(1950). Guide. Coronet.
- The weather.** 10 min, (1942). Encyclopaedia 551.5
Britannica Films.
- Winds and their causes.** 10 min, b&w or 551.5
color, (1948). Guide. Coronet.

Sound

- Command performance.** 22 min, (1942). The 789.91
recording of music. RCA-Victor, Institute
of Visual Training.
- Doppler effect.** 8 min, (1952). AAPT, Mc- 534
Graw-Hill.
- ***Fundamentals of acoustics.** 10 min, (1933). 534
Encyclopaedia Britannica Films.
- Instruments of the orchestra.** 20 min, (1945). 785.1
British Information Services, Eastin Pic-
tures.
- Introduction to sound.** 9 min, (1950). Guide. 534
G.B. Equipments, F993.
- Music in motion.** 20 min, color. (1953). Pho-
tography of sound. Bell System.
- Musical notes.** 12 min, (1950). J. A. Rank, 786
United World Films
- Nature of sound.** 10 min, b&w or color, 534
(1948). Guide. Coronet.
- Pen point percussion (b&w) and Loops** 792.93
(color). 7, 3 min, (1951, 1948). How hand-
drawn sound is made by Norman McLaren.
National Film Board (Canada), Inter-
national Film Bureau.
- ***Science in the orchestra: I. Hearing the** 785.1
orchestra, II. Exploring the instruments,
III. Looking at sounds, 34 min, (1952).
Realist Film Unit, British Information
Services, McGraw-Hill.
- Sound,** 12 min, (1939). Edited Picture Sys- 534
tem.
- Sound recording and reproduction.** 10 min, 621.345
(1943). Encyclopaedia Britannica Films.
- Sound waves.** 15 min, (1950). J. A. Rank, 534
United World Films.
- ***Sound waves and their sources.** 10 min, 534
(1933). Encyclopaedia Britannica Films.
- Sounds of music.** 10 min, b&w or color, 785.1
(1948). Guide. Coronet.
- Speed of sound.** 6 min, (1950). Guide. G.B. 534
Equipments F995.

Electricity and Magnetism

- Basic electricity.** 19 min, color, (1948). 537
USAF, United World Films.
- Basic electronics.** 17 min, color, (1948). 621.38
Walt Disney for USAF, AF 1-2136; United
World Films.
- Basic principles of frequency modulation.** 31 621.384
min, (1944). USA, TF 11-2069; United
World Films.
- Basic telephony.** 20 min, (1949). USA, TF 621.385
11-1553; United World Films.
- Capacitance.** 31 min, (1943). USN, MN- 621.38
1540-a; United World Films.
- ***The cathode ray tube—how it works.** 15 min, 621.342
(1943). USN MN 2104-a; United World
Films.
- Circuit testing with meters and multimeters** 621.328
I. Theory. 30 min, (1952). USA, TF 11-
1666; United World Films.
- Coaxial and microwave miracles.** 11 min, 621.384
(1952). Bell System.
- The diode.** 17 min, (1945). Guide. USOE, 621.342
United World Films.
- Effects of the ionosphere on radio wave** 621.384
propagation. 29 min, (1950). USA, TF
11-1632; United World Films.
- Electrical circuit faults.** 19 min, (1943). 621.328
Guide. USOE, United World Films.
- The electron—an introduction.** 16 min, 537
(1944). Guide. USOE, United World Films.
- Elementary electricity—current and electro-** 537
motive force. 11 min, (1945). USN, MN
1540-t; United World Films.
- Elements of electrical circuits.** 11 min, 537
(1943). Encyclopaedia Britannica Films.
- Elements of electricity.** 15 min, (1942). USA, 537
TF 11-622; United World Films.
- Exploring with x-rays.** 40 min, (1941). General 535.66
Electric.
- How television works.** 10 min, (1952). G.B. 621.388
Equipments, United World Films.
- In all weathers.** 26 min, (1950). Radar. 621.381
British Information Services.
- Magic in the air.** 9 min, (1942). Television. 621.388
General Motors.
- Measurement of electricity.** 10 min, b&w or 537
color, (1949). Guide. Coronet.
- Microwave oscillators.** 18 min, (1949). USA, 621.342
TF 11-1567; United World Films.
- Naturally it's FM.** 17 min, color, (1947). 621.384
General Electric.
- Ohm's law.** 19 min, (1943). USA, TF 11-1200; 537
United World Films.
- Primary cell.** 11 min, (1944). Encyclopaedia 621.35
Britannica Films.
- Principles of electricity.** 20 min, color, (1945). 537
General Electric S-2585.

- Principles of gas-filled tubes.** 15 min, (1945). 621.32
Guide. USOE, OE-353; United World Films.
- Radio antennas: creation and behavior of radio waves.** 12 min, (1943). USAF, TF 1-474; United World Films. 621.384
- Radio receivers: principles of radio receivers.** 17 min, (1945). USAF, TF 1-474; United World Films. 621.384
- Receiving radio messages.** 11 min, (1943). 621.384
Encyclopaedia Britannica Films.
- Röntgenstrahlen.** 20 min, (1937). Commentary in German. Ufa Kulturfilm. Museum of Modern Art. 535.66
- Rotating magnetic fields.** 13 min, (1945). 621.313
Guide. USOE, United World Films.
- Sending radio messages.** 10 min, (1943). 621.384
Encyclopaedia Britannica Films.
- Series and parallel circuits.** 11 min, (1943). 621.328
Encyclopaedia Britannica Films.
- Single-phase and polyphase circuits.** 17 min, (1945). Guide. USOE, United World Films. 621.313
- Taking the x out of x-rays.** 7 min, (1946). 535.66
Dr. W. D. Coolidge explains the x-ray tube. General Electric.
- Television: how it works!** 10 min, b&w or color, (1952). Guide. Coronet. 621.388
- Television receivers.** 7 min, (1953). McGraw-Hill. 621.388
- Television system.** 14 min, (1953). McGraw-Hill. 621.388
- †**Terra incognita.** 27 min, (1950). Electron microscopy. N. V. Philips' Gloeilampenfabrieken, Eindhoven, Netherlands, North American Philips. 535.8
- The thinnest slice.** 20 min, (1949). Electron 535.8
- The transistor.** 11 min, (1953). Bell System. 621.38
microscope specimen technique. University Southern California, RCA-Victor Division.
- Travelling electrical waves.** 50 min, si, (1936). 537
D-c waves on open line, short circuited, and loaded lines. Massachusetts Institute of Technology.
- ***The triode: amplification.** 14 min, (1945). 621.342
Guide. USOE, United World Films.
- Unseen worlds.** 10 min, (1941). Electron 535.8
microscopy. RCA-Victor Division, Institute of Visual Training.
- ***Vacuum tubes.** 11 min, (1943). Encyclopaedia Britannica Films. 621.342
- ***Vacuum tubes: electron theory and the diode tube.** 16 min, (1945). USAF, TF 1-470; United World Films. 621.342
- Volt ohmmeter operation.** 15 min, (1944). 621.3743
USN, MN 1540-s; United World Films.
- Voltaic cell, dry cell, and storage battery.** 18 min, (1943). USA, TF-11-1187; United World Films. 621.35
- Western relay.** 11 min, (1952). Western Electric. 621.385
- X-ray inspection.** 21 min, (1944). Guide. 535.66
USOE, United World Films.
- ### Light
- Colour.** 15 min, color, (1948). British Information Services, McGraw-Hill. 535.6
- Curves of color.** 10 min, (1941). General Electric. 535.6
- How motion pictures move and talk.** 10 min, (1939). United World Films. 621.345
- Introduction to optics.** 17 min, (1945). USN, MN 2449-a; United World Films. 535
- Lenses.** 10 min, (1950). J. A. Rank, United World Films. 535
- †**Light control through polarization.** 22 min, color, (1946); also 14 min, b&w, (1953). Polaroid Corp. 535.89
- Light is what you make it.** 10 min, color, (1946). Better Light Better Sight Bureau. 621.32
- Light waves and their uses.** 10 min, (1937). Encyclopaedia Britannica Films. 535
- Magic of fluorescence.** 17 min, color, (1945). General Electric. 621.32
- ***Measurement of the speed of light.** 8 min, (1952). AAPT, McGraw-Hill. 535
- Measurement with light waves.** 15 min, (1944). Guide. USOE, OE174; United World Films. 389
- ***Nature of color.** 10 min, color, (1947). Guide. Coronet. 535.6
- Nature of light.** 10 min, b&w or color, (1948). Guide. Coronet. 535
- Photography:** The basic camera, 15 min; Developing the negative, 16 min; Elementary optics in photography, 19 min; Light-Sensitive materials, 22 min, color; Navy photography in science, 28 min, color; Printing the positive, 19 min; Vectographs, Laboratory procedures, 21 min. USN, United World Films. 770
- Refraction.** 8 min, (1950). J. A. Rank, United World Films. 535.3
- Shadows and eclipses (reflection).** 10 min, (1950). J. A. Rank, United World Films. 535.3
- Spherical mirrors.** 13 min, (1950). J. A. Rank, United World Films. 535.3
- Use of the microscope.** 28 min, color, (1950). Scientific Film Co.
- ### Atomic & Nuclear Physics
- A for atom.** 15 min, color, (1953). General Electric. 539.76
- The atom and industry.** 12 min, (1952). Encyclopaedia Britannica Films. 539.76
- ***Atomic energy.** 16 min, (1947). Encyclopaedia Britannica Films. 539.76
- †**Atomic physics.** 80 min, (1947). J. Arthur Rank Organization, Ltd. (available, by purchase: United World Films; on loan, 539.76)

- U. S. Atomic Energy Commission.) Five parts: (1) The atomic theory; (2) Rays from atoms; (3) The nuclear structure of the atom; (4) Atom smashing; (5) Uranium fission: nuclear energy.
- Atomic power.** 19 min, (1946). March of Time, McGraw-Hill. 539.76
- Atoms at work** (Harwell Atomic Research Establishment). 11 min, (1952). British Information Services. 539.76
- Bikini—radiological laboratory.** 22 min, color, (1950). USAEC, United World Films. 539.76
- Canada's atom goes to work.** 11 min, (1952). National Film Board (Canada). 539.76
- First atomic pile.** 17 min, (1947), National Science Teachers Association, Teaching Films Custodians. 539.76
- Inside the atom.** 10 min, (1948). National Film Board, (Canada). 539.76
- Nuclear reactor.** 8 min, (1953). AAPT, McGraw-Hill. 539.76
- Operation crossroads.** 26 min, color, (1949). Two Bikini bomb tests. USN, MN-6817; United World Films. 623.451
- Operation Greenhouse.** 25 min, color, (1951). Nuclear weapons tests at Eniwetok, 1951. U. S. Atomic Energy Commission, United World Films. 623.451
- Operation Sandstone.** 18 min, color, (1951). Preparation for tests of atomic bombs at Eniwetok, 1948. U. S. Atomic Energy Commission, United World Films. 623.451
- Our atomic development.** 26 min, (1953). March of Time, McGraw-Hill. 539.76
- †Radioactivity: Laboratory Demonstrations.** 17 min, color, (1949). USA, PMF-5110; U. S. Army Central Film Libraries. 539.7
- †The Radioisotope series:** I. Fundamentals of radioactivity. 59 min, (1952) PMF-5145a; II. Properties of radiation. 68 min, (1952). PMF-5145b; III. Practical procedures of measurement. 48 min, (1952). PMF-5145c; V. Principles of radiological safety. 50 min, (1953). PMF-5145e; VI. Practice of radiological safety. 33 min, (1952). PMF-5145f. (Available, by purchase: United World Films; on loan, U. S. Army Central Film Libraries. A 17-page Guide to the training film series, and an instructor's guide for each film is available from Isotope Division, U. S. Atomic Energy Commission, Oak Ridge, Tenn.) 539.7
- Radioisotopes in general science.** 46 min, (1952). USA, PMF 5147-c; United World Films. 539.76
- Report on the atom.** 19 min, (1948). March of Time, McGraw-Hill. 539.76
- Target Nevada.** 16 min, color, (1953). U. S. Atomic Energy Commission, United World Films. 623.451
- Tale of two cities.** 20 min, (1946). USA, 940.54 ANSM 74; U. S. Army; United World Films.
- U-238 radioactive series.** min, (1953). AAPT, McGraw-Hill.
- Unlocking the atom.** 20 min, (1952). U. S. A condensation of **Atomic physics.** United World Films. 539.76

Distributors of the Films on this List

- Adel Division, General Metals Corporation, 1077 Van Owen Avenue, Burbank, California.
- Allis-Chalmers Manufacturing Company, Advertising and Public Relations Department, Milwaukee 1, Wisconsin.
- American Telephone and Telegraph Company, 208 West Washington St., Chicago 6, Illinois. (Bell System: Apply at your local telephone business office.)
- Atomic Scientists of Chicago, 1126 East 59th St., Chicago 37, Illinois.
- Bell System: Apply at your local telephone business office.
- Better Light Better Sight Bureau, 420 Lexington Avenue, New York 17, New York.
- Bray Studios Inc., 729 Seventh Avenue, New York 19, New York.
- British Information Services, 30 Rockefeller Plaza, New York 20, New York. (Films distributed by McGraw-Hill.)
- Canadian Film Board (National Film Board of Canada), 1270 Avenue of the Americas, New York 20, New York.
- Castle Films Division, United World Films Inc., 1445 Park Avenue, New York 29, New York.
- Coronet Instructional Films, Coronet Building, Chicago 1, Illinois.
- Eastin Pictures, Davenport, Iowa.
- Edited Picture System Inc., 165 West 46 St., New York 19, New York.
- Encyclopaedia Britannica Films Inc., 1150 Wilmette Avenue, Wilmette, Illinois.
- Franco-American Audio-Visual Distribution Center, Inc., 934 Fifth Avenue, New York 21, New York.
- Franklin Institute, Franklin School of Science and Arts, 251 South 22 Street, Philadelphia, Pennsylvania.
- General Electric Company, Film Distribution Section, Building 6, Room 210, 1 River Road, Schenectady 5, New York.
- General Motors Corporation, 3044 West Grand Blvd., Detroit 2, Michigan.
- Institute of Visual Training, 40 East 49 St., New York 17, New York.
- Johns-Manville Sales Corporation, 1617 Pennsylvania Blvd., Philadelphia 3, Pennsylvania.
- McGraw-Hill Book Company, Inc., Text-Film Department, 330 West 42 Street, New York 36, New York.
- March of Time Forum Edition, 369 Lexington Avenue, New York 17, New York. (Films distributed by McGraw-Hill Book Company, Inc., New York.)
- Massachusetts Institute of Technology (Office of John J. Rowlands), 69 Massachusetts Avenue, Cambridge 39, Massachusetts.

- Minneapolis-Honeywell Regulator Company, 4482 Wayne Avenue, Philadelphia 44, Pennsylvania.
- Modern Talking Picture Service Inc., New York Film Exchange, 30 Rockefeller Plaza, New York 20, New York.
- Museum of Modern Art Film Library, 11 West 53 Street, New York 19, New York.
- National Film Board of Canada, 1270 Avenue of the Americas, New York 20, New York.
- North American Philips Company, 750 South Fulton Avenue, Mt. Vernon, New York.
- Polaroid Corporation, Cambridge 39, Massachusetts.
- RCA-Victor Division, Camden 2, New Jersey.
- Scientific Film Company, 6804 Windsor Avenue, Berwyn, Ill.
- Sperry Gyroscope Company, Central Film Service, Great Neck, New York.
- Teaching Film Custodians, Inc., 25 West 43 Street, New York 36, New York.
- United World Films Inc., 1445 Park Avenue, New York 29, New York.
- U. S. Army Central Film Libraries. Address "Commanding General, Attention: Surgeon, Headquarters, . . ." First Army, Governors Island, New York 4, New York;
- Second Army, Fort George Meade, Maryland; Third Army, Fort McPherson, Atlanta, Georgia; Fourth Army, Fort Sam Houston, San Antonio, Texas; Fifth Army, Fort Sheridan, Chicago, Illinois; Sixth Army, Presidio of San Francisco, California. (Films available for purchase from United World Films.)
- U. S. Atomic Energy Commission, Public Information Section, 1901 Constitution Avenue, Washington 25, D. C.
- U. S. Bureau of Mines Experiment Station, 4800 Forbes Avenue, Pittsburgh 13, Pennsylvania.
- U. S. Department of Agriculture, Motion Picture Service, Office of Information, Washington 25, D. C.
- U. S. Navy, Motion Picture Section, Office of Public Information, Executive Office of the Secretary, Washington 25, D. C.
- Western Electric Company, Coordinator of College Relations, 195 Broadway, New York 7, New York.
- Westinghouse Electric Corporation, 401 Liberty Avenue, P. O. Box 2278, Pittsburgh 30, Pennsylvania. ("during the 1953-54 school year Westinghouse is discontinuing distribution of motion pictures. . .")
- Wright Aeronautical Corporation, Wood-Ridge, New Jersey.

The Ionization in Air Maintained by a Uniform Plane Circular Distribution of α Radioactivity*

R. W. NICHOLLS

University of Western Ontario, London, Ontario, Canada

(Received April 8, 1953)

The rate of ionization maintained in air in the region of a uniform plane circular distribution of α radioactivity has been computed numerically and is shown graphically. Such information is of interest as a lower limit upon the excitation rate maintained by the same source and is of use in the interpretation of spectroscopic observations in the excitation of radiation caused by the passage of positive ions (α particles in this case) through gases.

WHEN energetic positive ions traverse matter, the well-known phenomenon of ionization occurs along the paths of the particles. This phenomenon has been extensively investigated. An accompanying phenomenon which has not yet been so fully investigated is the feeble excitation of light along the track of a particle. This light is produced not only by collision phenomena involving the particle and the matter which it traverses, but also collision phenomena involving the secondary particles

(ion-pair members) produced along the track of the particle and the matter traversed by the particle. This effect is of course extensively used in scintillation counters. Studies of the light output produced in various solids and liquids have recently been reported¹ as have studies upon the chemical effects caused by the passage of high-energy ions through liquids.²

Very little quantitative work has, as yet, been undertaken upon the light emitted when en-

* The research reported in this paper has been sponsored by the Geophysics Research Directorate, Air Force Cambridge Research Center, Air Research and Development Command, under Contract AF 19(122)-470.

¹ E. W. T. Richards and J. F. I. Cole, *Nature* **167**, 286 (1951); Norman, Greenfield, and Kratz, *Nature* **171**, 487 (1953); F. W. Spiers, *Discussions Faraday Soc.*, **12**, 13 (1952).

² P. I. Dee and E. W. T. Richards, *Nature* **168**, 736 (1951).

ergetic positive ions pass through gases³ although some qualitative work has been reported on the feeble spectrum excited in the air by α particles in front of radium or polonium α -particle sources.⁴ There is thus a very fruitful field of investigation in the spectra excited by positive ion collisions with gaseous molecules which would have important application in the field of auroral physics because of recent work attributing part of the auroral spectrum to the result of collisions between solar protons and molecules in the upper atmosphere.⁵

It is with such applications in mind that the present work has been undertaken.

In some preliminary experimental work with an $f/7$ spectrograph, a 25-mc polonium source of α particles obtained from Atomic Energy of Canada, Ltd., mounted upon a small circular nickel plate, was collimated before the spectrograph and 300 hours exposure was required to record even the weakest spectra. The experimental work will be reported upon more fully later.

Because of the extremely low level of light intensity observed, the present discussion is an attempt to place a lower limit upon the excitation rate set up at points before the source of α particles.

THE EXCITATION RATE

The density of ion pairs produced along an α -particle track is well known, and the range vs ion-pair density data (Bragg curves), for α -particle tracks in air under standard conditions, are readily accessible for most common α emitters.⁶

Details of the accompanying milder phenomenon of excitation of gas molecules (in contrast to ionization) along the α -particle track, are less

well known. In the case of α particles from polonium, of maximum range 3.8 cm, the mean energy (averaged over the whole track) required to produce an ion pair is about 35 ev.⁷ N_2^+ ions, O_2^+ ions and electrons are the most probable ion-pair members. The respective ionization energies in these cases are 15.6 and 12.2 ev.⁸

Thus, 19 or more electron volts per collision are on the average dissipated in other ways, e.g., excitation or dissociation of molecules, or increasing the kinetic energy of members of the ion pairs.

While it is not the purpose of this paper to discuss in detail the complex problem of energy division between the possible processes, it will be noted that excitations not caused directly by the α particle⁹ will certainly be caused by the secondary collisions between the energetic ion-pair members and neutral gas molecules. In the case of N_2 , the readily excited energy states and the dissociation energy lie between 6 and 12 ev. The corresponding energy range in the case of molecular oxygen is 1–7 ev.⁸

In so far as collisions between air molecules and energetic electrons of the ion pairs are concerned, dissociation would be the most probable fate of O_2 molecules, and excitation the most probable fate of N_2 molecules. It will therefore be accepted as a reasonable hypothesis that there will be at least one molecular excitation phenomenon in the neighborhood of an ion pair for every such ion pair produced along the α -particle track. Thus, if the ionization rate produced in air at points before an α -particle source can be computed, this will be at least a lower limit upon the excitation rate maintained before the α -particle source.

COMPUTATION

Elementary analysis shows that the number I of ion pairs produced per second per cc at a general point P before an α -particle source of radius a on which N disintegrations occur per

³ H. S. W. Massey and E. H. S. Burhop, *Electronic and Ionic Impact Phenomena* (Oxford University Press, New York, 1952), p. 533; E. T. S. Appleyard, Proc. Roy. Soc. (London) A128, 330 (1930).

⁴ Sir Wm. and Lady Huggins, Proc. Roy. Soc. (London) A72, 196 (1903); A76, 488 (1905); A77, 130 (1906); B. Walter, Ann. Physik 17, 367 (1905); 20, 327 (1906); R. Pohl 17, 375 (1905); B. Walter and R. Pohl, Ann. Physik 18, 406 (1905); F. Himstedt and G. Meyer, Physik. Z. 6, 688 (1905); 7, 762 (1906); G. Ortner and S. Salim, Nature 169, 1060 (1952).

⁵ A. B. Meinel, Astrophys. J. 113, 50 (1951).

⁶ F. Rasetti, *Elements of Nuclear Physics* (Prentice Hall, Inc., New York, 1936), pp. 44–45; G. Stetter, Z. Physik 120, 639 (1943).

⁷ F. Rasetti, (reference 6); L. H. Gray, Proc. Cambridge Phil. Soc. 40, 72 (1944).

⁸ G. Herzberg, *Molecular Spectra and Molecular Structure* (D. Van Nostrand Company, Inc., New York, 1950), Vol. I. *Spectra of Diatomic Molecules*, second edition.

⁹ J. W. Frame, Proc. Cambridge Phil. Soc. 27, 511 (1931); N. F. Mott, Proc. Cambridge Phil. Soc. 27, 553 (1931).

sec per sq cm is given by

$$I = \frac{5N}{2\pi} \int_0^a \int_0^{2\pi} \frac{n(R) r dr d\theta}{R^2}, \quad (1)$$

provided the source is sufficiently thin that absorption may be neglected. In Eq. (1) R is the distance between the point P and the general area element $r d\theta dr$ on the circular α -particle source, and $n(R)$ is the number of ion pairs produced per mm of track at range R . This function is experimentally determined and has been tabulated.⁶ The α -particle source employed was of the standard size obtainable from Atomic Energy of Canada Ltd., viz., 1.15-cm diameter. For this thin source the absorption would be negligible for our purposes. Consequently the integral

$$I_1 = \int_0^{0.575} \int_0^{2\pi} \frac{n(R) r dr d\theta}{R^2} \quad (2)$$

is of specific interest here and has been computed numerically in the region $0 < x < 1.5$ cm; $0 < \rho < 1.5$ cm. Distance x is measured from the source along the axis of symmetry and ρ is measured from and at right angles to the axis of symmetry. The results obtained from this computation are shown in Fig. 1, where $I_1(x, \rho)$ is plotted as a function of ρ for the four values of x : 0.1, 0.5, 1.0, and 1.5 cm.

DISCUSSION

The curves in Fig. 1 clearly show the expected marked decrease in the rate of ion production, and thus optical excitation, as the distance from the axis of symmetry is increased. This decrease is more marked across planes closer to the source than those further from it. I_1 tends to a limiting value of about 10^8 ions per cc per disintegration at points on a hemisphere of about 1.5-cm radius about the center of the source.

From the point of view of light production accompanying this ionization, it would therefore

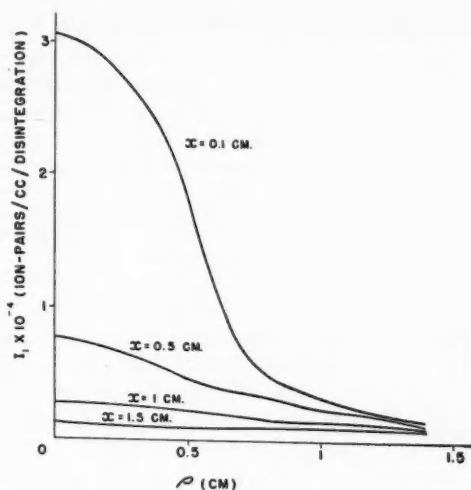


FIG. 1. Ionization density maintained before a polonium α -particle source of radius 1.15 cm. There are $5NI_1/2\pi$ ions produced per cc per sec at a point whose cylindrical polar coordinates relative to the center of a uniform circular plane distribution of polonium α radioactivity are (ρ, x) , when Na disintegrations occur per sq cm per sec at the source.

be expected, and is in fact observed, that the visible luminosity is largely limited to regions very close to the source. However, in that the much weaker luminosity from regions much further from the source is due to progressively less energetic collision phenomena than those at shorter distances from the source, it would be interesting to make a spectroscopic study of accompanying changes in the character of the emitted light at varying distances from the source. These light intensities are comparable to that of the light of the night sky (airglow), and thus a quantitative experimental study requires an instrument of high aperture ratio. The present calculations will be of use in such a study.

ACKNOWLEDGMENT

Grateful thanks are extended to Mrs. E. Parkma who performed nearly all the computations.

Some Characteristics of Military Research and Development*

DEAN E. WOOLDRIDGE

The Ramo-Wooldridge Corporation, Los Angeles, California

(Received July 20, 1953)

Military research and development possesses features of interest to physicists other than the employment opportunities provided by projects in this field. Military systems development work, in particular, affords technical problems of unusual variety, the solution of which frequently involves a degree of scientific teamwork not commonly encountered in other forms of industrial research and development.

RECENT surveys¹ indicate that approximately 40 percent of all physicists in this country now earn their living in whole or in part from military work of one form or another. There seems to be little reason to expect this proportion to decrease in the foreseeable future. Therefore, it is important for physicists to know what special characteristics, if any, differentiate militarily sponsored research and development from other kinds of professional work available to them. It is particularly important for teachers to be well informed on this subject, because of the influence they exert on the choice by students of a future career.

This paper treats one kind of industrial military development work that ranks high with respect to the number of physicists now engaged in it and gives promise of absorbing increasing numbers of physicists in the coming years—systems development work. In particular, it is pointed out that this kind of military development possesses two attributes that exert a major influence on the conditions under which the project physicists work. One of these attributes has to do with the variety and difficulty of the technical problems encountered by the physicist; the other pertains to the usual *teamwork* aspects of the scientific activities.

THE SPECIAL CHARACTERISTICS OF MILITARY SYSTEMS DEVELOPMENT

A systems development project is one in which a number of major complex components must simultaneously be developed to act to-

gether to perform some new or greatly improved operation, requiring that a considerable amount of development of various techniques beyond the present state of the art be accomplished in order to achieve the desired result. There is no better modern example of a military systems project than a guided missile development program, requiring as it does concurrent major advances in propulsion, aerodynamics, guidance, and control. Of course, systems projects have been generated by the military for many years; but in the past, because the technical advances involved have not been great, the military engineering and procurement organizations have usually been able to carry out the systems coordination themselves. Recently, however, the military people have been showing a steadily increasing appreciation of the fact that the technical content of the new tools of war is so large that responsibility for integrating complex military systems must more and more be put into the hands of outside contracting organizations.

As a consequence of this trend, developments have occurred in a number of industrial companies that are resulting in the establishment of organizational and technical project patterns of work quite different from the methods of industrial research and development commonly employed in the past. In order to illustrate what these changes are and how they came about, let us examine in some detail the nature of the technical problems involved in a specific military project, namely, the development of a guided missile. This will also help us to see the sort of organization and the environment in which the physicist must live if he is to participate actively in any similar program.

* Presented before the 15th Annual Colloquium of College Physicists, State University of Iowa, June 19, 1953.

¹ H. A. Barton, *Phys. Today* 5, 4-10 (1952); J. I. Mattill, *Phys. Today* 5, 14-18 (1952); Anon., *Phys. Today* 6, 20-21 (1953).

GUIDED MISSILES AS AN EXAMPLE OF SYSTEMS DEVELOPMENT

From many points of view, a guided missile does not differ essentially from military aircraft. Its purpose is to destroy some specific target. It travels through the air in getting to that target. Some missiles are intended to destroy surface targets and, therefore, belong in the same functional group as the bomber airplane. Other missiles are designed for use against enemy aircraft and, therefore, fall in the same general category as fighter airplanes. Also, with respect to the components of which they are built, guided missiles are like airplanes. In both types of device, a motor of some form is used to propel an air frame, which in turn is controlled in its path by movable surfaces that in airplanes are sometimes, and in missiles are always, assisted by autopilot equipment. In both cases, a warhead of some form is carried to perform the destruction, and navigation to the target is brought about by one form or another of guidance equipment.

Because of these similarities, one could easily come to an erroneous conclusion—that the development of a guided missile is essentially the same kind of problem as the development of a new airplane. In fact, this was the view held quite generally several years ago, and it has been responsible for a good deal of disappointment among many military and government people who thus thought that guided missiles in useful quantities and with useful qualities would be forthcoming with something like the same effort and on the same schedules as characterize the development and production of new airplanes. Recently, realization has grown that this conclusion was erroneous, but the reasons for the error are not generally understood. We shall see that the superficial comparison just recited fails to take into account two important differences. First, the technical problems confronting the guided missile designer are more difficult than those with which aircraft designers usually deal; second, the development of the missile is an integrated systems problem of a high order, by comparison with which the development of a new airplane is more like a component development problem.

To arrive at an understanding of the situation,

let us look at some of the differences between the problems confronting the designer of a fighter airplane and those confronting the designer of a particular type of guided missile, namely, the type intended to be launched against an enemy bomber plane.

VARIETY AND DIFFICULTY OF THE COMPONENTS PROBLEMS

Consider first some of the important design features of a modern jet interceptor plane. Such an airplane will weigh perhaps 15 000 lb. It is driven by a jet engine that typically has a thrust of several thousand pounds. The engine itself is rather massive and will run to several thousand pounds in weight and 50 to 100 ft³ in volume when installed. Consider next the guidance and control system of this fighter plane. To be an all-weather interceptor—the only type that can fairly be compared with a guided missile—it must be equipped with radar that can be used for spotting and then tracking the enemy bomber. This radar information is processed by a computer to provide steering instructions, which in turn are used in a servo loop that today typically includes a human pilot who flies the interceptor on a proper attack trajectory against the bomber and causes its armament to be fired against the bomber at the proper point. This guidance and control mechanism, including as it does radar equipment, computing equipment, and electromechanical actuators associated with the control surfaces of the airplane, runs to an installed weight of hundreds of pounds and a volume of tens of cubic feet, even without counting the human pilot.

Consider now the corresponding features of an antibomber guided missile. Such a missile is typically much smaller than a fighter plane; it may even have to be small enough to be carried by and launched from the fighter plane described. This small size of the typical missile is an important fact that must be borne in mind throughout all considerations of its components. Let us start with the missile motor. The antibomber missile is generally required to fly at speeds much higher than those reached by airplanes, and these speeds must be reached quickly after launching so that rapid overtaking

and destruction of the enemy target may be possible. Because of this combination of requirements, the motor installed in the small missile may have to provide more thrust than the jet engine of the very much larger fighter plane.

Considering next the missile air frame, we observe that the missile aerodynamicist must design an air frame and control surface mechanism for a range of supersonic speeds lying well beyond those of the fastest jet airplanes. At the same time, the missile must be designed to be exceedingly fast on its feet, in the sense that it must be able to outmaneuver any airplane that it is intended to attack; that is, it must be capable of attaining very high lateral accelerations that can be reached on demand with a minimum of time lag. Here again, these objectives must be accomplished within limitations of size and weight of the components that frequently lie well beyond the aircraft art.

We come now to the most difficult missile design problem—the guidance and control system. Here the designer must start by providing a mechanism in the missile that can supply accurate information as to where the target is and what it is doing. Such target reporting mechanism must be connected to a computing and data-processing apparatus that derives steering signals suitable for use by autopilot components which finally deflect the control surfaces. This is a short way of describing a tremendous technical task. Recall that, to accomplish these objectives in a fighter plane, use is made not only of radar, computer, and other electronic equipment weighing hundreds of pounds, but also of a human pilot who turns knobs, reads meters, makes judgments, and in general holds the whole operation together. Just the autopilot portion of the missile task is a formidable one. Of course, autopilots have been used in airplanes for a number of years. However, they have never been designed to control airplanes in more than gentle maneuvers. Moreover, the autopilot designer has allowed himself to use relatively long time constants in the mechanism so as to avoid otherwise difficult stability problems. In the guided missile, on the other hand, a very tight control loop is essential; the autopilot must permit full performance of an air frame that is much more maneuverable

and shorter in its response time than any aircraft that has yet been built. And here again the restriction on size of equipment is much more stringent than for the autopilots that perform much simpler duties in airplanes.

In approaching a solution of these difficult problems, the missile designer reasons the same way any physicist would. He observes that in a number of important directions—whether it is in propulsion, aerodynamics, or guidance and control—it is necessary to extend the art very considerably beyond where it stands today. He knows that all of the way through he is going to be tightly squeezed in his design considerations, with respect not only to size and weight, but also to the performance requirements that must be met by each of the components. Therefore, he starts his missile design operation by a careful study of what can be accomplished for each component by the most effective extension possible of *known* techniques.

It is characteristic of such studies that they rarely lead to a conclusion that is more than barely encouraging; a carefully balanced program of development *may* make it possible for the various component requirements to be met and to be packaged together in such a way that a successful missile may result. Here is an important difference between aircraft and guided missile development—the marginal possibility of achieving satisfactory missile performance. This difference actually is not surprising. It is simply a consequence of the fact that the guided missile field is new—new because it is only recently that motors, air frames, and guidance and control devices have developed to a point where it makes any sense at all to attempt what we are striving for today. Ten years from now, the situation may be different, but today the missile designer is pushing the state of the art in so many directions simultaneously that he cannot fall very short of realizing the ultimate in the performance of any of his major components without completely failing in his mission.

Although the necessity for *concurrent* advances in a number of technical directions is a dominant characteristic of guided missile work, it is to a considerable extent common to all major military systems development projects. This has an important effect on the environment offered to

physicists by such projects. They typically find available problems to work on covering a range of technical subjects of breadth not frequently encountered in other kinds of development activities. Certainly this is one of the most attractive features of military systems development work to the physicists involved.

UNUSUAL REQUIREMENT FOR CLOSE TECHNICAL COORDINATION

Another aspect of systems development that has a major bearing on the environment of the scientific personnel is the necessity for unusually close technical coordination of the activities of the component groups. Again, guided missile development is a good example to use in investigating this feature of military systems work.

To explore the coordination aspects of missile work, let us approach the missile performance problem from the point of view of the electronics guidance specialist. It is characteristic of missiles that the designer generally must work close to the noise level in whatever kind of guidance system he is using. This means, for example, that if there is some voltage that indicates to the missile that it should turn 2.0 degrees to the left, it is likely that this 2.0 degrees actually should have been, say, 1.5 or 2.5 degrees and that an excess of deficiency of voltage is present, arising from one of many sources of random error or "noise" in the fundamental guidance data. If the existence of such noise is ignored, then the random fluctuations in the signals that go to the missile actuators may cause the control surfaces to hop around rather wildly, resulting in bad missile performance. For example, the noise may overload the control system to such an extent that there is no proper response to the wanted signal portion of the voltage; or it may cause the missile, because of the added drag due to its frequent twistings and turnings, to slow down so much that the target is never reached. Therefore, those who design the guidance system must calculate these effects carefully so as to specify for the circuits the proper filters to reduce the noise to a tolerable level.

Suppose this has been done and that test missiles have been built. These are fired at targets, and telemetering information and motion picture data are obtained by means of which

the actual performance of the missile is analyzed. Suppose further that this analysis reveals the presence of more noise in the system than was predicted and allowed for in the design. This would not be surprising, because some of these calculations are concerned with complex physical phenomena that are not too clearly understood. Of course, there is no difficulty about simple circuit noise, which is easily dealt with. Rather, the noise of importance has to do with a combination of electromagnetic, electronic, aerodynamic, and mechanical factors, all of which contribute random errors in the steering of the missile.

If the calculations were faulty and the noise is appreciably higher than allowed for, what must be done about it? The answer might seem obvious—simply increase the filtering to remove more of the high-frequency fluctuations. However, the physicist will realize that the more filtering he puts into the circuit, the more time delay there will be between the introduction of basic target information into the input end of the missile circuits and the transmission to the actuators of the resulting missile response commands. Low-pass filtering is always accompanied by time delay. But an appreciable increase in the response time of the missile to target information could easily result in inadequate tactical performance. So what is to be done? Depending on the type of missile, there may be any one of several solutions to the problem. For one type, the answer might be to make it possible for the missile to perform more violently when it does finally respond. Let us assume that this is the solution adopted in the example under study. In such a case, the missile design modification will indeed include the interposition of additional filtering, to satisfy the guidance experts. But then a problem is handed to the aerodynamicists, who must modify the air frame to provide higher lateral accelerations. This requires enlarging the wings and the control surfaces—a modification not to be lightly undertaken in a supersonic speed device. Assume, however, that the aerodynamics redesign is successfully accomplished. Next it is probably necessary to call upon the designers of the various components that fill the inside of the missile to make changes in the shapes or positions

of these components so as to restore the center-of-pressure to center-of-gravity relationship that is so important for stability and control, and which was probably upset by the air-frame modification. Finally, it must be remembered that the enlarged wings will add drag and diminish the range of the missile. Thus, it is not unlikely that the propulsion experts will have to get in the act to provide a new motor capable of higher thrust.

The point here is evident. We started with a problem that seemed to lie strictly within the domain of the electronics guidance specialist and have found that its solution has ramifications that affect all parts of the guided missile. In brief, for the successful development of a guided missile, it must be looked upon as a single entity—a single system.

An exceedingly close interrelationship must be maintained at all times among the groups who are assigned responsibility for the various technical subdivisions of the guided missile. It is not possible to go nearly so far in the direction of dividing design responsibility among different component experts as in the airplane field. Again, this is a consequence of the fact that useful guided missiles are just barely possible with the state of the component art that exists today. In the example just cited, the discovery of unexpectedly high noise in the guidance system could have been solved as a purely local electronic design problem if the original air-frame design had provided for lateral acceleration considerably in excess of that thought to be necessary—as a design safety factor. But such an approach cannot be used today, for such safety factors, if applied to all components, would certainly result in an inoperative or tactically useless weapon.

It is important not to underestimate the magnitude of the technical coordination task. Because of the multiplicity and complexity of the component problems, a typical guided missile development project requires teams made up of hundreds of scientists and engineers, supplemented by additional hundreds of helpers, technicians, draftsmen, mechanics, and the like. If we think in terms of a project team of a thousand people or more, all engaged in the development of a single type of guided missile,

we will not be far wrong. When one considers that the missile must be designed as a single entity with close technical relationships among all of these people, the magnitude of the coordination problem becomes apparent. In many respects, the management problems in such military systems work are as formidable as the technical ones. It is reasonably certain that the time required to complete a successful development of a guided missile depends just as markedly on the effectiveness of the organizational patterns and coordination procedures in channeling the effort into paths leading toward the same goal as it does upon the quality of the individuals who make up the team.

Although organizational and management problems are of direct concern to those interested in science administration (in fact, it would be hard to find a field that offers more challenge and stimulus to the science administrator), the large majority of physicists are not primarily occupied with such matters. Of more importance to this discussion, therefore, is the effect that these special large-scale coordination problems have on the environment and the conditions within which the physicists on such projects must work. As can be imagined, these considerations are not trivial ones. For instance, it obviously would be difficult for a large-scale military systems project organization to make effective use of the individual genius who does not work well with other people. There is no better advice that the teacher can give to a graduating student of this type than to avoid such employment. Practically without exception, such men are unsuccessful in systems development work; usually they are unhappy, make their associates unhappy, and either quit or are discharged within a short time.

Another example of the result of the requirement for closely integrating many technical teams working on various phases of the same major problem is the central role that must be played by groups of *analytical* physicists. A major military systems project, if it is properly organized, will achieve the required technical coordination primarily through the workings of such a strong central group of competent analytical people, who at every stage perform the necessary calculations and analyses that are

required to determine the design parameters of the various components of the system in such a way that the over-all objectives of the project can best be met. It is such a group that would be assigned the task of tracing the effects of a change in noise filtering in the guidance system of a missile on the design parameters of the other components of the system, in the example used earlier. In short, the central analysis group is usually the key to the technical integration of a major systems project; it receives the results of all pertinent experiments and component analyses and makes sure that the technical objectives of each of the component groups working on the project are mutually consistent and, when changed, are consistently changed with respect to one another.

However, it would be a mistake to assume that all analytically inclined physicists are gathered together in such central teams. The problems encountered in some of the components sections, while more special in character, frequently require the continued participation of unusually well-trained analytical physicists. The design of an air frame of minimum drag, high stability, and high maneuverability that will operate at three times the speed of sound, for example, is a components problem in the terminology of this paper, but it is still challenging enough to suit most aerodynamically trained physicists and engineers.

Other examples of the effects of the coordination requirement on the environment of the project physicists arise out of the problems of communication among the scientists and engineers of the various technical groups. There must be frequent conferences and meetings for the purpose of interchanging information and permitting joint participation in the formulation of plans and decisions. Also, time schedules must be established for the various subgroups to try to meet, for there is no other way of determining whether the balance of effort among the various technical problems is correct and of insuring consistency in the dates of availability of the various interdependent components or technical data.

Many more examples could be provided of the manner in which the professional life of the physicists on a military systems development

project is influenced by the requirement for unusually close technical coordination of the work of large numbers of scientists and engineers. Although the details of the methods and procedures employed to effect such coordination differ somewhat among industrial organizations, in every case one of the most prevailing influences is the *scientific teamwork* aspect of the activity.

CONCLUSION

No attempt has been made here to present a complete picture of the advantages and disadvantages to physicists of involvement in military research and development. Not only has the discussion been limited to one kind of military work—systems development—but even with respect to this limited topic no account has been taken of the existence of certain well-known disadvantages to the scientific project people, arising out of the basically destructive character of the work, the necessity for the observance of security regulations, administrative and red-tape problems in working with the government, and the like. The justification is that the case against working in military fields has frequently and adequately been presented; there has probably been too little effort exerted by physicists active in this kind of work to describe some of the desirable characteristics of military development.

Whether we like it or not, many physicists are going to be involved in military work for a long time to come. Nothing short of the attainment of real world harmony and the almost complete elimination of our military establishments can prevent this. The military have now discovered science, and scientists themselves, by the very effectiveness of their efforts, have made it impossible to escape continued involvement. But the prospect need not be an entirely gloomy one, particularly not to those physicists who enjoy *applied* science. Much satisfaction can be derived from participation in programs that afford not only a wide selection of challenging technical problems, but also concurrent effort on these problems that often result in the inventive ideas of the physicists being carried through to useful end products much more rapidly than is typically the case in nonmilitary

work. There is also the stimulus provided by the interplay of ideas with other scientists and engineers on the research team whose skills and training cover a wide range of technical fields. Finally, there is satisfaction in the realization that it is only through such large-scale, highly

coordinated effort that success can be achieved in bringing reality to the fantastic new weapons and techniques of push-button warfare that the magazine and newspaper writers have been proclaiming for so long, but that still remain just a little beyond the reach of the physicist.

Laboratory Teaching in General Education Courses*†

CLEMENT L. HENSHAW
Colgate University, Hamilton, New York
(Received June 15, 1953)

The extent to which laboratory activities are used in general education science courses, and the nature of these activities, have been investigated in the course of visits to 16 institutions and by reference to catalogs of 13 additional liberal arts colleges. Laboratory was found to be fairly regularly used in about two-thirds of the physical science courses, and in a still larger fraction of the biological science courses. Probably less than ten percent of the students at these institutions earn the B.A. degree without a substantial laboratory experience of some kind. Significant attitudes, techniques and experiments in practice at a number of institutions are reported, and an appeal is made for many more to be published. As a result of the study, a set of ideals for "discovery-type" laboratory activities for general education courses is suggested.

I. INTRODUCTION

ALTHOUGH the basic college courses in physics and in all other sciences traditionally have included regular laboratory work, the practice is far from universal in science courses for general education. A number of the latter seem to be conducted successfully with little or no laboratory activity. The question of the purpose of laboratory instruction immediately arises. Is it important only for the science specialist? Does it contribute to objectives of teaching in general education? If so, how? Or are there more effective substitutes for the time and expense of laboratory instruction? While no unanimity exists, new viewpoints on some of these questions are developing in several places in the

country,¹ and some of those which have come to the attention of the writer are worth reporting.

II. EVOLUTION OF GENERAL EDUCATION SCIENCE COURSES

In certain discussions, education is often divided into two categories, *special* and *general*. By special education is meant that experience which is directed at preparation for a career, for earning a living, and it is with various aspects of special education that most of the activities of physics teachers are concerned. By general education, in contrast, is meant that experience which prepares a person, beyond merely sustaining himself, to live with maximum social and personal effectiveness in the contemporary world. There is no sharp cleavage between general and special education, nor is general education something new which has recently been discovered. In fact, it has many ideals and practices long associated with *liberal* education.

* An amplification of one of four invited papers presented in a Round Table on Undergraduate Laboratory Teaching at the Annual Meeting of the American Association of Physics Teachers held at Harvard University, January 22-24, 1953.

† The remarks in this paper upon specific programs are derived in nearly all cases from observations made by the author while visiting the institutions named during his tenure of a Faculty Fellowship granted by the Fund for the Advancement of Education of the Ford Foundation. The author is solely responsible for the conclusions expressed.

¹ G. M. Dunning, *Sci. Teacher* **18**, 85 (1951). H. Kruglak, *Am. J. Phys.* **19**, 223 (1951). W. E. Morrell, *J. Chem. Educ.* **30**, 80 (1953). G. E. Owen, *Am. J. Phys.* **17**, 270 (1949); **19**, 381 (1951). H. Kruglak, *Accent on Teaching: Experiments in General Education*, edited by S. J. French (Harper and Brothers, New York, 1954), Chap. 12.

That science makes an important contribution to a person's *general* education has long been accepted in America. Two centuries ago it was in the form of mathematics and astronomy, then natural philosophy and natural history. By the middle of the 19th century the separate sciences had appeared, but even at the peak of the free elective system President Eliot of Harvard acknowledged that "no required studies now remain except the writing of English, the elements of French or German, and a few lectures on chemistry or physics."²

One might say that over a century ago science courses in most colleges in this country were taught *solely* to fill the needs of general education. It is true that the first technical school, Rensselaer Polytechnic Institute, was founded at Troy, New York in 1824, and during the first half of the century rapid technical advances occurred in industry, communication, and transportation. These were largely the result of empirical ingenuity. The realization that scientific knowledge was rich with practical applications did not begin to grow until after the Civil War.

Several changes in the character of science teaching resulted. The "science specialist" evolved and efforts were made to improve his preparation. In the course of these efforts, it was discovered that science and technical students gained a clearer understanding in science, as well as practical skill, if they had the opportunity to manipulate equipment themselves instead of just watching demonstrations by the professor. Finally, as our comprehension of the natural world extended at an accelerated rate, the field of science enlarged and proliferated almost explosively. For some time, specialists and nonspecialists alike, found themselves in the same introductory courses. But accumulating knowledge, combined with the increased mathematical difficulty of the specialist courses, led to the provision of similar but easier and less comprehensive courses for other students.

In the past 25 years, there has been a growing belief that the general education in science needed by the nonspecialist is not met adequately

by an imitation of the beginning courses for specialists, which are designed primarily as preparation for courses at higher levels. Much of the factual information and technical skills are of limited value to a person not continuing in science. The first break away from this pattern occurred with the use of the *survey* approach, in which topics were drawn from several related fields such as physics, chemistry, astronomy, and geology. These survey courses were generally conducted without laboratory, and lectures, with demonstrations and other visual aids, were depended on to provide both interest and understanding.

In spite of its ability to bridge the artificial cleavages between fields of science, the survey method has been criticized in the past decade as being too superficial and emphasizing mostly a mass of factual information. Its critics claim that little understanding of the nature of the scientific enterprise is gained in this manner. As substitutes we have seen a variety of approaches tried, the block-and-gap method in a single science or in a group of related sciences, the straight historical approach, the use of case histories, the problem approach, the careful study of original documents, and several hybrids of these. Although these various methods differ in the details of their objectives, there is a general effort (1) to get away from dogmatic teaching, (2) to convey some understanding of how science knowledge is gained and what it means, and (3) to develop some aspects of a scientific attitude in the students. It is interesting that the exponents of some of these approaches disparage the value of individual student laboratory, while others attach great importance to it.

III. TO HAVE OR NOT TO HAVE LABORATORY

Some architects of general education science courses belittle the contribution of laboratory activities of the type commonly used in specialist courses. The objectives of this work, they claim, are out of harmony with their general education objectives. For example, traditional laboratory trains students for special skills. In introductory physics, students are taught techniques of handling basic instruments they will use later: calipers, balances, timing devices, ammeters, voltmeters, oscilloscopes and the like. They are

²C. F. Thwing, *A History of Higher Education in America* (D. Appleton and Company, New York, 1906), p. 437.

also trained in special methods of determining the precision of experimental measurements. Such techniques are of little value to the non-specialist.

Traditional laboratory is notably discordant with general education in the purposes of many of its exercises. In the field of physics, "experiments," so called, are planned which deductively verify relations that previously have been derived or dogmatically asserted by the textbook or lecturer. Students manipulate carefully designed equipment to *confirm*: the laws of equilibrium of forces, of the pendulum, of Hooke, Newton, Boyle, Charles, Ohm, Faraday, or Snell. Teachers of general courses, on the other hand, see little value in the student's showing that his laboratory equipment conforms to stated laws. They are anxious to have him understand how these laws—or any laws in science—are arrived at in the first place.

One also finds in many specialist-course laboratories that the planner has an incorrigible desire for the student to get the most complete and precise results in the limited time available, with the consequence that techniques become more involved and instructions more rigid and detailed. Such a condition tempts a critic to use the epithet, "Cookbook!" Strict control of operations, they say, prevents a thoughtful approach to a problem.

Other objections to laboratory are raised in terms of subject matter. If one intends to have laboratory, conventions of scheduling dictate that it come at regular intervals, commonly once a week. But many general education science courses draw blocks of material from the areas of astronomy, geology, meteorology, or atomic physics, which are not the conventional laboratory sciences of physics and chemistry. The objectors claim that it would be impossible to find suitable activities in these areas that would maintain a continuous laboratory program. Relevant projects would be trivial, or they would involve costly equipment beyond the students' comprehension, or require manipulative techniques beyond the students' powers. They don't consider a laboratory activity of the right kind as worthless, they merely claim that while it is essential for the specialists, it is not worth

the cost in time, space, and staff for the non-specialist.

Proponents of the inclusion of laboratory activity in general education courses take their position, at least partially, as an article of faith.¹ They have a conviction that science is incomplete without some direct sense experience. The writers of the Harvard report state their opinion of the place of laboratory as follows:

The scientist ensures himself, when he can, the proper circumstances for pursuing his inquiry by ordering the conditions of the natural events himself. This is the point of scientific experiment. By this means matters may be so arranged as to yield an unequivocal answer to a highly specific question concerning the real world. Such regulations of the system under regard is beyond the powers of students in other areas of reality. It is this constant appeal to things as they are which makes the direct experience of the field and laboratory essential in scientific education. Needless to say, this is so only to the degree that work in the field or laboratory is designed not merely to keep students busy or develop technical proficiency, but to provide directly the materials of scientific argument and the tests of scientific hypothesis.²

In describing a proposed physical science course they say, in part:

(The laboratory) should be planned to illustrate the methods by which physical problems are solved. Every effort should be made to convey these as genuine experiences . . . in the scientific solution of material problems.³

One can recognize that the above statements are high-sounding phrases, fine in principle, but do not deal with the realities of laboratory operation. An enthusiast for general education could still maintain that the methods of solving problems, and the subordination of theory to fact, can be conveyed more fully through the treatment of case histories and by reference to the results of laboratory work of others. What evidence is there that this sort of thing can actually be accomplished in a laboratory activity within the capacity of the nonspecialists?

Here one may turn to the testimony of those who have struggled to create valid laboratory experiences:

I feel strongly that laboratory work should be involved in general education physical science courses, and I agree that they should not be routinized or perfunctory. At

¹ The Harvard Report, *General Education in a Free Society* (Harvard University Press, Cambridge, 1945), p. 151.

² Reference 3, p. 228.

Oklahoma A. and M. and at Chicago, we worked considerably with projects in which we tried to make the student aware of a problem or a question answerable by laboratory methods. We then asked the student to propose methods whereby he might seek answers to these questions, promising to provide him, within reason, with whatever equipment he might require. I must confess that this pedagogic attitude sometimes was abandoned as the press of duties encroached upon planning time. Nevertheless, when the attitude was carefully adhered to, and when the project was well carried out, it was the most gratifying sort of laboratory teaching experience which I have ever had.⁵

In the words of Eric Rogers at Princeton University:

Laboratories need not be (routine and boring). They need experiments which have some flavor of genuine investigation, not ones which just give training in techniques or servile "verification" of known laws. Tests of laws can be good indeed, but need a different attitude. The student needs time to try his experiments his own way, repeating them if he wishes. Then students and teacher need time for discussion, to see what can be drawn from the experiment and why. Most experiments should be simple and should be allowed to run for two or more weeks. . . . With fewer experiments than beta (the standard introductory course for specialists), the equipment need not be expensive, but the staffing raises serious problems because the instructors need to be keen, patient, sympathetic, well-informed scientists who have the aims of the course at heart. . . . Although sympathetic staffs are essential they need not be numerous. Relieved from pressure to "get the right answer" in a hurry, students are not lazy, but take more care and like wrestling with their problems without much help.⁶

These staunch defenders of the laboratory claim that in no other way can one so effectively achieve an active participation on the part of the student. He is not just the passive observer; he is the *performer*. He must make decisions and must act, not only with his brain, but with his hand and eye. Participation brings real experience into the learning process. They also claim that the laboratory is unequalled for convincing a student that science, as a body of knowledge and a system of theories, is ultimately based upon observables. The relations between fact and inference, hypothesis and definition, are more vividly and directly conveyed in terms of the specific and tangible problems of the laboratory, than in historical discussions or the con-

sideration of a grandiose problem in the classroom.

IV. CURRENT USAGE OF LABORATORY IN GENERAL EDUCATION COURSES

In the particular group of 16 institutions visited,⁷ it was found that about two-thirds of the general education physical science courses include fairly regular laboratory of some kind. While less attention was paid to the biological sciences, the impression was gained that a still larger fraction of the general education courses in biology made some use of laboratory. This small sample is thus consistent with much more extensive studies which have reported 60 percent and 80 percent usage of laboratory in physical science and biological science general education courses, respectively.⁸

Out of curiosity, the sample of 16 was extended to 29 by examining the available catalogs of a representative selection of liberal arts colleges.⁹ In some instances, confusion was discovered about what constitutes a "general education" course in science. It is not uncommon for a liberal arts college to stipulate certain requirements "to provide for a student's general education." These requirements may be nothing more than a distribution of introductory courses in each of several fields. In such cases, there is little evidence to show that the science courses have a general education character as described in Sec. II. Often one cannot decide without observing the course in operation. It appears that in fewer than one third of the entire group is the science requirement for the B.A. degree satisfied by nothing more than one or two years of some standard laboratory science. The remainder are making positive efforts to introduce general education aspects of science through specially designed courses which either are necessary and sufficient for the science requirement, or which fulfill the requirement when added to, or substituted for, a standard introductory course.

⁷ Amherst, Chicago, Cornell (N.Y.), DePauw, Drake, Harvard, Haverford, Kalamazoo, Michigan State, Minnesota, Princeton, Swarthmore, Wesleyan (Conn.), Wisconsin, Wooster, and Yale.

⁸ Bullington, *Science Educ.* 35, 92 (1951); Entrikin, *J. Chem. Educ.* 28, 274 (1951).

⁹ Allegheny, Bates, Bucknell, Carleton, Colby, Denison, Oberlin, Pomona, Redlands, Reed, Ripon, Union (N.Y.), and Williams.

⁵ Malcolm Correll, DePauw University, private communication.

⁶ *Science in General Education*, edited by Earl J. McGrath (William C. Brown Company, Dubuque, Iowa, 1948), pp. 18-19.

Total science requirements for the B.A. degree are difficult to summarize because so many combinations occur. But if attention is confined to the laboratory aspect it is fairly clear, if all the factors are considered, that probably less than ten percent of all of the students at these institutions graduate without having had a substantial laboratory experience in some science.

It is evident that the preponderant opinion of these college faculties is that science laboratory is an important component in preparation for the B.A. degree. Furthermore, a considerable number of them believe that a student's general education can be provided more effectively by courses different from those designed for the specialists. In many of these general education science courses, it has been demonstrated that laboratory activities can be designed which contribute to the objectives of the course and avoid most of the criticisms of traditional laboratory referred to earlier. Special efforts in this direction are more noticeable in physical science than in biological science courses. It may be that conventional laboratory work in biology is more easily adapted to a general education approach because the observing can be handled by the nondirective technique.

V. EXAMPLES OF GENERAL EDUCATION LABORATORY ACTIVITIES

Some of the laboratory projects that have been worked out in different places are quite intriguing. At Minnesota, Haym Kruglak early in the quarter makes this assignment, "Go out for the next three nights and observe the moon. Write a brief report on your observations." The discussion that follows, he says, is quite lively. A week later each student constructs an astrolabe from doweling, mailing tubes, and screws. This he uses to determine the altitude and azimuth of celestial objects. Kruglak even succeeded in developing a technique for each student to build a simple diffusion cloud chamber from a coffee can, peanut butter jar, cotton, dry ice, and alcohol. Some routine procedures like a chemical analysis can be described in cookbook style, he claims, if it serves as a bridge across an unimportant aspect of the problem. It is the complete blueprint of a laboratory exercise

leading to a result the student knows in advance, that Kruglak strongly objects to.

In an effort to illustrate the application of science to daily living, as well as provide practice in scientific thinking, Kruglak has drawn upon advertisements, particularly those which suggest a consumer test, for group experiments.¹⁰ The students first discuss the proposed test to discover its good and poor features, then they design and carry out an investigation of the claim which avoids, as far as possible, the weaknesses they recognized. The results often do not confirm the contention of the advertiser.

Eric Rogers believes that maximum freedom and simple equipment should be used whenever possible. He supplies his students with assorted balls, string, supports, and simple magnifiers for their wrist watches, and asks them to investigate the properties of a simple pendulum. They frequently accomplish little during the first afternoon and ask for more time. Even after two periods their results will probably not be as well organized or complete as those turned in by a student given detailed instructions, but Rogers believes they have acquired something else more valuable, a feeling for scientific investigation, and a sense of discovery or creativity at their own level.¹¹

Rogers has found that the nondirective technique which works well in experimental studies of pendulums or lenses, for example, needs some modification in other areas. When investigating electrical circuits the potential hazards to meters are discussed in advance, and students must have their circuits checked before connecting to a battery. But the criterion for approval is merely whether or not the instruments are in jeopardy. If a voltmeter and ammeter are in series with an unknown resistance, the student is allowed to go ahead and find his own way out of the difficulty. Rather than being cautioned to avoid extreme or damaging conditions, students are encouraged to extend the range of study as far as they are curious. Such test specimens as resistances and springs are considered "expendables," like chemical reagents.

With a small group, a teacher, if he is willing

¹⁰ H. Kruglak, *Sci. Teacher* **18**, 206 (1951); *J. Chem. Educ.* (to be published).

¹¹ See G. W. Stewart, *Am. J. Phys.* **13**, 291 (1945).

to spend the time, can encourage experiments that may involve apparatus construction. Ian Barbour at Kalamazoo College assigned his class the task of determining the temperature of dry ice in alcohol. Within a two-week period each group of students was to design and assemble its own apparatus (with assistance, if glass blowing was involved) and they had to use different designs. When they were finished they reported their results to the class with the merits and faults of the method. Aroused student curiosity, keen motivation, and valuable teaching opportunities are claimed for this project.¹²

At the University of Colorado they capitalize upon their local situation, where fairly high elevations are accessible in the mountains to the west, with a broad nearly level plain to the east. For one laboratory project, they station students at several different elevations and have them time the appearance of the sun over the horizon at sunrise. From these observations a rough figure for the radius of the earth can be determined.

Although there is no regular laboratory in the general education course at Colgate University,¹³ occasional laboratory activities occur. While studying, the Carolina Bays students are taken on a 20-mile field trip. Instead of merely pointing out important geological features of the landscape, the trip is used to study a parallel problem of the origin of the many undrained depressions of the region.

Several institutions, particularly those which have recently built science buildings, are discarding any distinction between laboratory and discussion sessions of their groups. They meet for periods of one, one and one-half, or two hours in rooms adapted to both laboratory and discussion, and carry on whatever activity is most relevant at that point in the problem they are considering. This procedure removes the objection to laboratory that good projects are not available at a uniform frequency in the course.

Michigan State College at the moment is in the throes of replacing separate year courses in physical and biological sciences in the Basic College by a single year course in natural

science. Their staff has a strong conviction of the importance of laboratory in such courses. An interesting approach to laboratory which developed in the biological science course¹⁴ is now being tried in some physical science experiments. The manual supplies fairly detailed sheets for each experiment which begin by suggesting that the student perform certain operations. Then he is asked what relation he sees between his observations, or what hypothesis he can propose to account for them, and he writes his responses in blanks provided. By careful planning of these sheets, a student can be carried through an experiment in a way which encourages him to think for himself and still arrive at some significant results. Some critics of their approach approve of the general idea but dislike the extent to which the student's next moves and thoughts are suggested. Doubtless this could be reduced in an institution in which registration for such courses does not run to several thousand.

The College of the University of Chicago has a three-year sequence in natural science which is required of every student who cannot demonstrate equivalent proficiency. Great stress is placed in these courses on the thoughtful discussion of important historical documents. Yet these discussions are paralleled by appropriate laboratory sessions with which the staff is still experimenting.

At Amherst College directions for experiments are given only in general terms. For example, as the second part of an experiment in magnetism the students are asked to design and carry out "an experiment to investigate the torque which is exerted on a coil of wire when it is placed in a magnetic field and a current is passed through it." The only directions supplied are the following:

First decide what aspects you are going to investigate and describe your plan in your notebook. Then try the attack you outlined. It is very likely that there will be something wrong or inadequate about your initial approach. Identify your difficulty if such is the case, describe it in your notebook, and then outline your improved plan of attack. Carefully sketch each experiment and describe all observations. Keep on doing this until you have successfully determined the direction of the torque on the coil under a variety of conditions and interpreted the results in terms of the ideas of forces on moving charges

¹² I. G. Barbour and R. O. Kerman, *Am. J. Phys.* **20**, 493 (1952).

¹³ See reference 6, pp. 39-57.

¹⁴ Lewis, *Sci. Educ.* **31**, 14, 157 (1947).

(developed in the class work). The most careful record of observations, including discussion of false starts, is the one deserving the highest grade.¹⁶

One might have supposed from the strong endorsement given to laboratory by the Harvard Report on General Education that the courses there¹⁶ would be using it. In the two biological science courses there are fairly regular laboratories, but in the four physical science courses there is virtually nothing. When visitors ask what caused the reversal of position, the answer, quite sincerely, is that when those concerned faced the problem of setting up a new set of required science courses for nine to twelve hundred Harvard and Radcliffe freshmen, the practical aspects of space and staff time, at least in the physical sciences, seemed to be more costly than they felt laboratory was worth. This decision to omit laboratory in the physical science courses was influenced in no small degree by the nature of the courses developed. Those science teachers who were courageous enough to forsake some of their specialties and pioneer in this new venture happened to be men who were most interested in the theoretical side of science, or in its historical development. It was only natural to expect them to design courses in harmony with their personal talents.

There is evidence¹⁷ to indicate that these courses are well respected by students, particularly for the high quality of the lectures, which come three times a week. The section meetings (once a week) conducted by graduate assistants were often rated as dull, though perhaps acknowledged as needed for getting a good mark. In view of the practice at Harvard of offering

¹⁶ It should be pointed out that the Amherst general education course is unique in that it is required of *all* freshmen and includes enough basic physics, chemistry, and mathematics that the specialist courses build upon it, counting it roughly equivalent to one-half year each of physics and mathematics. The students spend two hours per week in math sections, two hours first semester and three second semester in physics lectures, and each section meets one afternoon each week, alternately, for discussion or laboratory work. A total of eight semester-hour credits is assigned to the course. The subject matter is largely mechanics in the first semester and structure of matter second semester. Elements of the calculus are applied after they have been introduced, but strong emphasis is placed upon verbal statements of physical principles and their application. A symbolic presentation of a formula followed by numerical substitution receives virtually no credit, even with the correct answer. Many freshmen have found this to be a rugged experience but over 90 percent survive.

¹⁷ See reference 6, pp. 89-109.

¹⁸ *Confidential Guide to Freshmen Courses* (The Harvard Crimson, 1952), twenty-seventh edition, pp. 33-37.

several alternative science courses to fulfill the requirement, it seems obvious to the casual observer that a better balance of aims of general education would be obtained, if at least one of these courses were to include some carefully organized laboratory. The all-too-common inability of graduate students in physics to acquire the attitude and spirit of these general courses indicates, however, that laboratories under their direction might fall far short of acceptable goals, supporting the present contention that good lectures are more valuable than poor laboratories.

Institutions, like Princeton and Minnesota, that do use graduate assistants for conducting general education laboratories, have found it essential to provide for careful supervision as well as initial indoctrination. Other institutions, like Chicago and Michigan State, believe that no one below the rank of instructor should teach any part of these courses. It is clear that the success of general education courses in science, or probably any field, depends largely upon the calibre of the teaching staff.

VI. CONCLUSIONS

The writer's thoughts on this problem of the place of laboratory in general education can be summarized as follows: he is convinced that there are now good general education science courses taught without laboratory, but few would not be improved with the introduction of a limited number of carefully designed laboratory activities. Ideally, it seems, such activities will contribute most to common objectives of general education:

1. When they deal, as often as possible, with relations or problem situations for which the student does not know the answer in advance.
2. When they call for as much planning, initiative, and creative thinking as possible on the part of the student, that is, provide him an opportunity to *experiment* in the true sense of the word.
3. When they are integrated as well as possible with the objectives and subject matter of the course.
4. When they are carried on in an atmosphere in which curiosity is the motivation, rather than academic penalty.

5. When they are conducted by experienced teachers who are in sympathy with the aims of general education and have a sincere interest in developing mature minds.

Such ideals are hard to attain. They require much imagination and planning on the part of the instructor, particularly if a graduated program beginning with simple problem situations and progressing into more elaborate discovery projects, is to be achieved. Many science teachers are enthusiastic about these new approaches to laboratory but feel handicapped by a long conditioning to the traditional point of view. The urgent need at present is not new manuals or treatises on method, but a multitude of brief, candid reports of such projects by individual teachers, imperfect though these efforts may seem. This writer takes the liberty of appealing for an increased number of such reports to be published in the *American Journal of Physics*.

This student centered "discovery" approach to laboratory, as it might be called, has arisen as teachers have tried to avoid the routine and perfunctory aspects of the traditional approach, when designing laboratory activities for general

education courses. There is increasing evidence¹⁸ that such changes in viewpoint are occurring in the introductory courses for specialists, a trend which many teachers will applaud. The writer can see valid reasons for the specialist courses and those for general education to differ in both objectives and conduct. But he feels that, besides the discovery-type laboratory, there are other valuable features of general education science courses which the great majority of science specialists do not now experience. It is tempting to hope that the customers of our specialist courses in the future may draw still further dividends from the general education movement in science.

VII. ACKNOWLEDGMENT

The writer wishes to acknowledge his indebtedness to many persons in the institutions visited, too numerous to mention individually, for most of the suggestions and ideas presented above, and for their encouragement to assemble them here in this form.

¹⁸ Miller, Am. J. Phys. 19, 190 (1951); Pippert and Roller, Am. J. Phys. 20, 467 (1952); Straley, Fussler, and Shearin, Am. J. Phys. 19, 313 (1951); Wall, Kruglak, and Trainor, Am. J. Phys. 19, 546 (1951); Kruglak, Am. J. Phys. 21, 14 (1953); V. E. Eaton, "Aims and methods in the undergraduate physics laboratory," Am. J. Phys. (in press).

Practical Aids for Physics Teachers

The *American Journal of Physics* has introduced a new feature under the title, "Practical Aids for Physics Teachers." It has already begun in a small way, but we hope that it will grow to occupy several pages in every monthly issue.

The aids that go under such a title must be contributed; they cannot be prepared in the Editor's office. Material of the following kind would be most welcome:

A. Examination papers that seem to have been successful ones at all levels of difficulty, even into the beginning graduate level, but especially at the elementary level.

B. Problems, particularly at the elementary level; also at the upper undergraduate level. If problems are very difficult, solutions should be provided. They will be printed with the problems.

C. Useful analogies that may be used to clarify some of the basic concepts of physics; or useful illustrations of fundamental ideas.

D. Neat ideas for lecture demonstrations.

E. Useful material in books, magazines or trade journals that are not likely to come to the attention of the average teacher of physics but which offer illuminating discussions of simple matters. Give the

reference completely and specifically: summarize in one or two sentences.

The contributor of each item will be recognized in the following way: At the end of each item (or it may be at the beginning) will be placed the following statement: *Contributed by John Doe, Penpoint College.* This form will be used if the contributor wishes to be identified.

Should the contributor not wish to be identified, a form such as the following should be employed, for example, in the case of a contributed examination paper: *Used in a first-year physics course for liberal arts students in a private college of approximately 800 total enrollment.* Or else it might run: *Used in a physics course for sophomore engineering students in a state university of total enrollment 15 000.*

Now you see what I mean. Please send me contributions. The only stipulation that I must make is that everything must be typed double-spaced. When I say everything, I mean *everything*. Otherwise, there is no room for editorial markings. A mimeographed copy that is already double-spaced and of good quality would be acceptable.

This is your Journal, your Association. Won't you help?

THOMAS H. OSGOOD
Editor

Reproductions of Prints, Drawings, and Paintings of Interest in the History of Physics.

58. *Retombera-t-il?*

E. C. WATSON

California Institute of Technology, Pasadena, California

(Received July 27, 1953)

The experiment of firing a cannon ball vertically upward to see whether it would fall back into the mouth of the cannon was carried out in 1634 by Mersenne and Petit. A vignette from Varignon's *Nouvelles conjectures sur la pesanteur*, which portrays this experiment, is here reproduced.

THE question of whether a projectile shot vertically upward from the earth will fall back to its original place is an old and interesting one. It, together with the closely allied problem of whether a sphere falling from rest through still air follows the vertical indicated by a plumb bob, has engaged the attention of some of the greatest scientific minds—ARISTOTLE, COPERNICUS, TYCHO BRAHE, GALILEO, DESCARTES, NEWTON, HOOKE, LA PLACE, and GAUSS, to mention only a few of the most famous names. It even stimulated NEWTON to a renewed consideration of planetary orbits without which the *Principia* might never have been written.

The history of this famous problem affords an interesting example of the interplay between theory and experiment leading to the gradual discovery and clarification of general principles which characterizes physical science at its best. It has been given some detail by ANGUS ARMITAGE¹ and so will not be repeated here. Instead we shall merely reproduce a vignette from VARIGNON'S *Nouvelles conjectures sur la pesanteur* (Paris, 1690), which portrays the experiment

carried out in 1634 by MARIN MERSENNE, the French Minorite friar, friend and scientific correspondent of DESCARTES, who did so much to spread the ideas of GALILEO throughout Europe. It shows a cannon being fired vertically upward while two observers, one a clerical figure (MERSENNE), the other military (PETIT, a commissioner of fortifications), attempt to follow visually the flight of the projectile.

The experiment in this form was repeated several times during the seventeenth century, but the results in general were inconclusive, because of the difficulty of firing in a truly vertical direction. MERSENNE and PETIT apparently expected the projectiles to strike the ground far to the westward because of the rotation of the earth, but they failed to find them at all, and DESCARTES when consulted about the matter expressed the opinion that at great height projectiles would lose their weight and go off into space never to return. Others having a better appreciation of the principle of inertia expected the projectile to fall back into the mouth of the cannon. Actually, however, they should fall a small distance to the west, because by the law of conservation of areas their tangential velocity must decrease with height.²

The problem of where a body dropped from a great height will land is a much more subtle one, as NEWTON well realized. He failed to give the correct solution himself and it remained for GAUSS³ to do so. A completely satisfactory experimental verification still seems to be lacking, however, the last and most elaborate attempt being that of E. H. HALL at Harvard in 1902.⁴

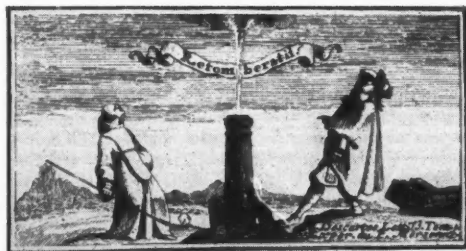


FIG. 1. Vignette from Varignon's *Nouvelles conjectures sur la pesanteur* (Paris, 1690).

¹ A. Armitage, *Ann Sci* 5, 342 (1947). See also E. H. Hall, *Phys. Rev.* 17, 179 (1903), and F. Cajori, *A History of Physics* (Macmillan Company, New York, 1929), pp. 81-82.

² See W. Ferrel, *A Popular Treatise on the Winds* (New York, 1893), p. 88.

³ Gauss, *Werke*, (Göttingen, 1867), Vol. V, pp. 495-503. See also W. H. Roever, *Wash. Univ. Studies* 3, 153-168 (1916).

⁴ E. H. Hall, *Phys. Rev.* 17, 179-190, 245-254 (1903).

Wave and Inertial Properties of Matter*

RICHARD SCHLEGEL

Michigan State College, East Lansing, Michigan

(Received July 27, 1953)

The inertial properties of matter as expressed in Newton's second law, and the mass-energy equivalence, are derived as consequences of the deBroglie equation, the Lorentz transformations, and the energy-frequency relation, $E = h\nu$. Inertia is then presented as an effect resulting from the frequency and hence energy changes with motion that result from the special relativistic time transformation and apply to mass because of the energy-mass equivalence. The deBroglie wave equation and Newton's second law are shown to be equivalent with respect to a postulate set for the deductive presentation of wave and inertial properties of matter; the deBroglie equation and Newton's second law may therefore be regarded as having the same physical content.

THE inertial properties of matter are usually presented in physics courses as primary properties. Thus, Newton's second law, which states essential inertial characteristics of matter, is customarily not derived from other laws but is stated as a first principle of mechanics. There is such an abundance of evidence in support of Newton's second law that it is easily and naturally presented as a primary axiom.

The reflective student and teacher must, however, often ponder the question, "What is the origin of matter's inertial resistance to motion?" The answer to this question, if it is to be other than simply "the inherent nature of matter," must proceed by relating inertial properties to other aspects of the physical world. In this paper we shall present one way of relating inertia to other physical properties, using ideas that are well established in present-day physics. Specifically, we will see that inertial properties may be derived from the wave properties of matter and radiation. It will then be possible to discuss inertia as other than an inherent property of matter that is without relation to other fundamental properties.

WAVE PROPERTY EQUATIONS

The wave property of matter is described by the deBroglie equation, which we write in its simple one-dimensional form, $\lambda = h/mv$, where λ is the observed wavelength associated with particles of mass m moving with velocity v , and

h is Planck's constant. We will take the deBroglie equation as an empirical equation, which relates the quantities λ , h , v , and a parameter m , and as an empirical equation it will form one of the assumptions of our derivation of inertial properties.

It may be noted that deBroglie's equation gives a method for determining mass values which is altogether different from the usual methods of making mass determinations. Masses are usually determined by their weight or by their resistance to velocity change, that is, by their gravitational or inertial properties. But in principle, at least, the individual m value of a stream of like particles moving with a uniform velocity could also be determined from wavelength and velocity observations by the use of the deBroglie equation. The m so determined would of course be just a parameter since the deBroglie wavelength observations alone would not tell us anything about the inertial properties associated with mass.

We will require two other relations that are descriptive of wave properties, in addition to the deBroglie equation: the relation $E = h\nu$, which associates a frequency ν with any elementary particle or photon of energy E , and the Lorentz transformation equations of special relativity theory. Specifically, the transformation equations will be used in the form

$$t' = (t - vx/c^2) / (1 - v^2/c^2)^{1/2} \quad (1)$$

In this equation the time interval t for events with a space interval x , where t and x are for an observer in a rest coordinate system Σ , is related

* Presented in part at the American Physical Society meeting, Rochester, New York, June 18-20, 1953.

to the time interval t' that will be observed from a coordinate system Σ' which is moving with a velocity $v=v_x$ with respect to Σ . Quantity c is the velocity of light.

The $E=h\nu$ equation, which was introduced into physics by Planck and Einstein for radiation and first applied to matter by deBroglie, may be regarded today as one of the fundamental postulates of physics. The Lorentz transformation equations rest essentially on measurements of the velocity of light. Neither the Lorentz equations nor the energy-frequency relation involve any reference to inertial or momentum considerations.

THE ENERGY-MASS AND MASS-VELOCITY RELATIONS

The energy-mass equation, which states that a mass m is equivalent to an energy $E=mc^2$, is implied by our three wave property equations. In order to make the derivation, we first note that Eq. (1) requires that a process which has a zero time interval, $\delta t'=0$, associated with it in the Σ' -coordinate system will present a time interval $\delta t=(v/c^2)\delta x$ for an observer in the Σ -coordinate system. This means that two events which are simultaneous to a Σ' observer and are separated by a space interval δx to the Σ observer will have a time interval $\delta t=(v/c^2)\delta x$ between them to the Σ observer. Or, in a word, a moving space interval implies a relativistic time interval.

We consider a deBroglie wave process associated with a particle of observed mass m which is at rest in Σ' . The process is observed from Σ , with respect to which Σ' moves with a velocity $v=v_x$, and for the Σ observer the process is characterized by a frequency ν and wavelength λ . Observations are made, as by an interference method, over some interval δx . In δx there are $\delta x/\lambda$ waves, and also, since there is a time interval $\delta t=(v/c^2)\delta x$ associated with δx , there must be $\nu\delta t=\nu(v/c^2)\delta x$ waves associated with the interval. Since the number of waves in δx must be the same whether determined in terms of wavelength or frequency,

$$\delta x/\lambda = \nu(v/c^2)\delta x, \quad \text{or,} \quad \nu\lambda = c^2/v, \quad (2)$$

which is the usual equation for the velocity of a deBroglie wave.

Substitution into Eq. (2) of h/mv for λ , as given by the deBroglie equation, followed by substitution of E/h for ν , from $E=h\nu$, leads at once to

$$E=mc^2. \quad (3)$$

It is to be expected that Eq. (3) would follow from using the deBroglie equation with the Lorentz transformations and the energy-frequency relation since in effect deBroglie's original derivation of his equation was based just on the energy-mass equivalence and the Lorentz transformations, with the all-important addition by him of an ascription of frequency $E/h=mc^2/h$ to any mass m .¹ It is of course our assumption of the deBroglie equation as an empirical equation which makes possible the derivation of the energy-mass equation without any assumption of inertial properties of matter. Customarily, the energy-mass equivalence is derived either: (1) from the mass-velocity relation, which in turn has been gained from the assumption of momentum conservation and the Lorentz transformations;² or (2) from the momentum content of electromagnetic radiation, as required by Maxwell's equations, and conservation of momentum or energy.³

With Eqs. (1) and (3), we may readily obtain the relativistic mass-velocity equation. We consider a particle of mass m which is at rest in the coordinate system Σ , and has a frequency ν associated with it. For an observer in a system Σ' , to whom the particle is moving with a velocity $v=v_x$, the associated frequency is ν' . The frequencies ν and ν' are in the same ratio as the t and t' intervals of Eq. (1), since "clock rates" of a process are taken as being proportional to the frequency of oscillations or other cycles of the process. We write Eq. (1), for time and space intervals, in the form

$$\delta t' = (\delta t - (v/c^2)\delta x) / (1 - v^2/c^2)^{1/2}.$$

But for a particle of mass m , which is at rest in Σ , $\delta x=0$, and $\delta t' = \delta t / (1 - v^2/c^2)^{1/2}$. Hence, the relation between frequencies as observed from

¹ L. deBroglie, *Ann. Physik* **3**, 22 (1925).

² See, e.g., R. C. Tolman, *Relativity, Thermodynamics, and Cosmology* (Oxford University Press, London, 1934), Sec. 27.

³ See, e.g., M. Born, *Atomic Physics* (G. E. Stechert and Co., New York, 1936), Appendix VII.

Σ' and Σ is

$$\nu' = \nu / (1 - v^2/c^2)^{1/2}. \quad (4)$$

It may be noted, should the reader be puzzled by the fact that Eq. (4) gives the higher frequency and "faster clock reading" for the moving observer, that one gains from Eq. (1) the contrary result, $\nu' = \nu(1 - v^2/c^2)^{1/2}$, for observations on electromagnetic radiation. In such cases, δx cannot of course be zero in Σ or any other coordinate system. The predicted *decrease* of frequency for an observer who is moving with respect to the source of electromagnetic radiation implies a shift to the red end of the spectrum and has been observed in the so-called "transverse Doppler effect."

The energy-mass-frequency relations, $E = mc^2 = h\nu$, when applied to Eq. (4), give us at once the relativistic mass-velocity relation,

$$m' = m / (1 - v^2/c^2)^{1/2}, \quad (5)$$

where m and m' are fixed in coordinate systems that are, respectively, at rest and moving with a velocity v with respect to the observer. Equation (5) has, then, been derived from our three wave property equations: (1) the deBroglie equation; (2) the energy-frequency relation; and (3) the Lorentz transformation relation between space and time intervals.

FORCE AND ACCELERATION

With Eq. (5), we have found the essential description of inertial properties. For, if we multiply the equation by c^2 and use $E = mc^2$, we may write it as $E' = E / (1 - v^2/c^2)^{1/2}$. The equation then states that in gaining velocity a mass gains energy—hence, mass is apparently not "indifferent to motion," but has the property that energy expenditure is required in giving it motion. The law of motion is, however, not customarily stated simply in terms of the energy that is associated with a given velocity but rather in terms of the force that is associated with a given acceleration.

In order to gain the usual form of the law of motion, we write, from Eq. (5), as is customary,

$$K.E. = m'c^2 - mc^2 = mc^2(1/(1 - v^2/c^2)^{1/2} - 1), \quad (6)$$

where $K.E.$ is the complete, "relativistic"

kinetic energy of a mass m that has been accelerated to a velocity v .

As is well known, a series expansion of the right-hand side of Eq. (6) gives

$$K.E. = \frac{1}{2}mv^2 + \frac{3}{8}mv^4/c^2 + \dots \text{terms of higher powers,}$$

so that for the case where $v \ll c$ we can write

$$K.E. = \frac{1}{2}mv^2. \quad (7)$$

We next introduce an arbitrary force F_x in the x dimension by the definition, $K.E. = \int_0^x F_x dx$, where velocity \dot{x} is specified as equal to zero at $x=0$. We equate the two expressions for $K.E.$ that are given by Eq. (7) and the defining expression for force:

$$\int_0^x F_x dx = \frac{1}{2}m\dot{x}^2.$$

Differentiation with respect to t gives:

$$d/dt \left\{ \int_0^x F_x dx \right\} = d/dx \left\{ \int_0^x F_x dx \right\} dx/dt = m\dot{x}\ddot{x},$$

or,

$$F_x \dot{x} = m\dot{x}\ddot{x},$$

or,

$$F_x = m\ddot{x}, \quad (8)$$

which is the usual expression of Newton's second law in one dimension. Similar expressions may be found in the y and z dimensions. Multiplication by unit vectors and addition will of course give the vector equation,

$$\mathbf{F} = m\mathbf{a},$$

where \mathbf{a} is the acceleration vector.

For the relativistic equation between force and acceleration, we proceed from Eq. (6) in a similar manner. We again introduce a force F_x by the definition $\int_0^x F_x dx = K.E.$, $\dot{x}=0$ at $x=0$. Then equating expressions for $K.E.$ gives $\int_0^x F_x dx = mc^2(1/(1 - v^2/c^2)^{1/2} - 1)$, where motion is taken to be in the x direction, $v = \dot{x}$. Time differentiation gives:

$$\begin{aligned} d/dt \left\{ \int_0^x F_x dx \right\} &= d/dx \left\{ \int_0^x F_x dx \right\} dx/dt \\ &= mc^2 d/dt \{ 1/(1 - v^2/c^2)^{1/2} - 1 \}, \end{aligned}$$

or,

$$F_x \dot{x} = m \dot{x} \dot{x} / (1 - v^2/c^2)^{3/2},$$

or,

$$F_x = m \ddot{x} / (1 - v^2/c^2)^{3/2},$$

or,

$$F_x = d/dt \{ m \dot{x} / (1 - v^2/c^2)^{3/2} \} = d/dt (m' \dot{x}).$$

Similar expressions may be found in the y and z dimensions, so that we may finally write the usual relativistic expression, $\mathbf{F} = d\mathbf{p}/dt$, where \mathbf{p} signifies momentum, $m'\mathbf{v}$.

It should be pointed out that in introducing force by the definition which we have given we are not surreptitiously bringing in any momentum or inertial properties of matter. Energy would result from the application of a force through a distance, as say against a spring, even though no forces at all were needed to give acceleration to a mass. We do say, in our development, that the energy which results from applying a single force through a distance to a mass particle is kinetic energy, but we do this without concern for what is the inertial resistance to force. If, for example, the law of motion were $F_x = m \ddot{x}$, we would still write $\int_0^x F_x dx = \text{kinetic energy}$, since we define force as any "thing" which produces energy in acting through a distance. The essential element for gaining the proper laws of motion is not in the definition of the relation between force and energy but is in the relation between energy and velocity, Eq. (6). Throughout this paper, we shall consider force, as defined in this section, to have been introduced as part of the term energy, which has of course been introduced in the $E = h\nu$ equation.

INERTIA AND SPACE-TIME RELATIONS

The derivation of inertial properties from wave properties gives a clear illustration of the fact that inertia is not an inherent property of matter, independent of relations between matter and space-time. Going back through the derivation, we see that Eqs. (5) and (6) rest essentially on the energy-mass equivalence and on Eq. (4), which gives the change of frequency with velocity. Equation (4), in turn, results from the relation between time intervals for rest and moving observers that is required by the Lorentz transformation, Eq. (1). We can qualitatively describe the origin of inertia as follows: the

relativistic time change with velocity requires that a moving elementary process will have a higher frequency than one at rest. Since such a frequency is related to energy by the equation $E = h\nu$, more energy will be associated with a moving than a rest process. Since energy is equivalent to mass, a moving mass particle will be required to have greater energy than a rest particle. The amount of energy required for a given velocity is found to be just that amount which requires the usual relation between force and resulting velocity change per unit time.

If there were no change of the elementary frequency $\nu = E/h$ with velocity, there would be no energy change with velocity and hence no inertial resistance to motion. We see, therefore, that inertia is properly a "relativistic effect," not just for very high velocities where the energy of motion E is sufficiently great that E/c^2 implies measurable mass value changes, but for any velocity whatsoever. For it is the change, accompanying motion, in the time variable that gives rise to the frequency change with motion; and that frequency change in turn requires energy change with motion and hence application of force to produce the energy.

The mass-energy equivalence, in our presentation, will appear only with the addition of the deBroglie equation to the energy-frequency and the Lorentz relations. The close relation between the deBroglie equation and energy-mass equivalence can be qualitatively expressed by pointing out that the Lorentz transformations require a change with motion in the frequency associated with an energy; if mass is equivalent to energy, the frequency change will result in a wavelength change for the energy-related oscillations associated with the mass. Or, put another way, if there were no energy associated with mass, we would not expect to find for mass the frequency $\nu = E/h$ which gives rise to the deBroglie waves of mass particles.

In showing a relation between inertial properties and the space-time properties that are given by the Lorentz transformations and deBroglie wave properties of space-time, we emphasize, in terms of elementary physical relations, what has for several decades been commonplace in more advanced physical theory. One of the primary tenets of the general theory of relativity

has been that matter and space-time structure are dependent each upon the other. And it has been shown that the general relativity field equations—which relate space-time structure to matter in that space-time—do require the law of motion, in effect, Newton's second law, without additional assumption.⁴ The general relativity derivation, although long and complex, is of course much more fundamental than the derivation of the law of motion which we have given in this paper; whereas our derivation proceeds from several empirical relations for wave properties, the general relativity derivation shows that the primary equations for the structure of a space-time field do require Newton's second law.

EQUIVALENCE OF THE EQUATIONS OF NEWTON AND DEBROGLIE

Finally, we consider two possibilities for the postulates in a deductive presentation of the wave and inertial properties of matter and point out an interesting consequence of those possibilities.

Newton's second law does essentially describe the inertial properties of matter; with conservation of momentum, which is a direct consequence of the second law, plus the Lorentz transformations and the energy-frequency relation, the mass-energy equivalence and the deBroglie equation for wave properties are customarily derived.^{1,2} Therefore, we could take Newton's second law, $E = h\nu$, and the Lorentz equations as a set of postulates for a description of wave and inertial properties. The development that we have given shows, however, that we could describe the same properties by taking the same postulates, except with the deBroglie equation substituted for Newton's second law. This second set of postulates, that is, the deBroglie equation, $E = h\nu$, and the Lorentz transformations, would be more unified than the first set in that the postulates would all describe wave properties whereas the postulates of the first set directly refer both to wave and inertial properties. It might be objected that Newton's second law would be preferable as a postulate to the

deBroglie equation because force-acceleration situations which confirm the law are so much more readily observed than are deBroglie waves. This objection alone, however, need not be a compelling one; in chemistry, for example, the highly sophisticated nature of evidence for the atomic theory does not prevent that theory from being a good starting point in the presentation of elementary chemistry.

The similar roles of Newton's second law and the deBroglie equation in the description of wave and inertial properties may be more explicitly indicated as follows. As has been stated, the Lorentz transformation equations, which we will indicate as 'a', the energy-frequency relation, indicated as 'b', and Newton's second law, indicated as 'c', will give us the deBroglie equation, indicated as 'c'', as well as the mass-energy equivalence and various other consequences of wave and inertial properties, indicated as 'G'. Using an arrow to indicate 'implies' we can then write

$$(a, b, c) \rightarrow (G, c'). \quad (9)$$

We have seen, however, that the deBroglie equation, along with a and b , will also give us Newton's second law, as well as the mass-energy equivalence and, of course, various consequences of the deBroglie equation and the second law. Hence, we can write

$$(a, b, c') \rightarrow (G, c). \quad (10)$$

We see that c' and c are interchangeable as assumption and consequence in the relations (9) and (10). This means that c' and c are equivalent with respect to the postulate set (a, b) . Hence, having taken a and b as fundamental postulates, we make the same addition to our deductive system whether we add c' or c as a third postulate; or we can properly say that, with respect to the set (a, b) , c' and c say the same things about the physical world, although in different terms. Since the set (a, b) , with the addition of c' or c , does describe the essential wave and inertial properties of matter, it is legitimate to say that Newton's second law and the deBroglie equation are equivalent statements of certain fundamental physical properties, the first giving the statement from a particle point of view and

⁴ A. Einstein and L. Infeld, *Can. J. Math.* 1, 209 (1949); also P. G. Bergmann, *An Introduction to the Theory of Relativity* (Prentice-Hall, Inc., New York, 1947), Chap. XV.

second expressing the same physical content from a wave point of view.

Instead of writing deBroglie's equation as $\lambda = h/mv$, we may substitute E/ν for h , and then write the equation in the form

$$\nu(mv) = E/\lambda. \quad (11)$$

Comparing Eq. (11) with Newton's law, written in the form

$$d(mv)/dt = F, \quad (12)$$

we see that in both equations a quantity of momentum per unit time is equated to a quantity with force dimensions. But whereas Eq. (12) is written in terms that are appropriate for a particle description, Eq. (11) is written in wave terms. The postulates (a, b) give a physically correct transformation between the two equations.

I am grateful to Professors Harry Bendler and L. M. Kelly, both of Michigan State College, for conversations on the topics of this paper.

Quantum Mechanics and Thermodynamic Continuity. II

ALFRED LANDÉ

The Ohio State University, Columbus, Ohio

(Received August 4, 1953)

The program of deducing quantum mechanics from the postulate of thermodynamic continuity is extended to embrace the principles of symmetry and quantum statistics, in particular the rule of conservation of symmetry type when a system of N particles in resonance interaction is augmented by one more particle. The special quantitative definition of quantum conjugacy contained in the commutation rule of Born and in the Schrödinger replacement of the momentum p by a differential operator may be replaced by a general qualitative physical definition: " p and q are conjugate observables in a mechanical system when the manifold of q reactions of the system uniquely determines the manifold of its p reactions." When interpreted in terms of superposition mechanics this definition leads to the rules of Born and Schrödinger and from there to quantum dynamics.

1. INTRODUCTION

IN a recent investigation¹ it was shown that certain general principles of quantum mechanics, originally found from microphysical evidence and condensed in the Born-Heisenberg-Jordan and Schrödinger theory, may be deduced as direct consequences of the general postulate of *continuity of cause and effect*. More specifically, since the classical distinction between two states of a mechanical system as "either like or unlike" leads to the discontinuity paradox of Gibbs for the diffusion entropy of two like as against two unlike gases, one is compelled to admit fractional likeness values between states of gas particles and of systems in general. When unlikeness is identified with separability, and likeness with inseparability, then fractional likeness must imply some sort of fractional separability. This

leads, as we saw in Part I, to the arrangement of all states of a system into sets of mutually orthogonal or completely separable states, the set $A_1 A_2 \dots$ and the set $B_1 B_2 \dots$, and so forth, with a web of fractional likenesses, $q(A_k, B_j) < 1$, connecting the members of different sets. The same fraction $q(A_k, B_j)$ also indicates the relative intensity in the splitting of the state A_k into component states B_j , or the probability of an individual particle (system) in the state A_k to turn to the state B_j and then be passed by a B_j -selecting device. When looking for a self-consistent correlation law between the probabilities q so that the elements of the matrix $q(A, B)$ together with those of $q(B, C)$ will determine the elements of the matrix $q(A, C)$, one finds only one simple and natural mathematical solution of this correlation problem, namely, the rule

$$\psi_{km} = \sum_j \psi_{kj} \psi_{jm}, \quad (1)$$

¹ "Quantum mechanics and thermodynamic continuity," Am. J. Phys. 20, 353 (1952); also Phys. Rev. 87, 267 (1952).

where the quantities ψ are defined as complex amplitudes

$$\psi_{km} = \sqrt{(q_{km})} \exp(i\phi_{km}) = \overline{\psi_{mk}}. \quad (1')$$

It is well known how the superposition rule (1) leads further to a general superposition or matrix formalism for the eigenvalues and transition values of various observable quantities, with ψ itself playing the role of the "observable unity." In particular, one has the transformation rule

$$a_{ij} = \sum_k \psi_{ik} a_k \psi_{kj}, \quad (2)$$

where a_k is an eigenvalue of the observable a in the state A_k and $a_{ij} = \bar{a}_{ji}$ is the transition value (matrix element) of a between two states B_i and C_j . The ground structure of quantum mechanics, including the superposition formalism, is thus obtainable as a consequence of the postulate of thermodynamic continuity together with that of self-consistency.

Yet quantum mechanics has a superstructure built on additional rules of a more specific character. We refer to the Schrödinger rule of identifying a momentum p with the differential operator $(\hbar/i)\partial/\partial q$, and the Born commutation rule, $qp - pq = i\hbar$, both rules being equivalent definitions of *canonical conjugacy* between a coordinate q and a momentum p . Secondly we refer to the *symmetry rules* for systems of identical particles, rules which are fundamental for the structure of atoms, molecules, and the physical qualities of matter in bulk. It is our intention to show in the following pages that conjugacy and symmetry rules can be deduced from simple general postulates. The rule " ψ is either symmetric or antisymmetric," i.e., the division of all particles into the Bose and the Fermi kind, will be shown to be a direct consequence of the postulate of entropy continuity. The quantitative qp rules of Schrödinger and Max Born can be replaced by a less stringent general definition of conjugacy not requiring a special quantitative relation between q and p , but introducing conjugacy only by the following qualitative statement: " q and p are defined as conjugate observables of a system when the q reaction of the system uniquely determines its p reaction." A clarification of this somewhat enigmatic statement will be given in Sec. 3. At

any rate, the present investigation tries to round out our effort of presenting the concepts and rules of quantum mechanics as direct consequences of simple and plausible assumptions about the working of nature, rather than as an assortment of mysterious rules about noncommutative algebra, etc., applied to complex imaginary quantities.

2. SYMMETRY PRINCIPLES AND QUANTUM STATISTICS

A statistical theory of thermodynamic phenomena is possible only when the many parts or particles of the thermodynamic macro-system are supposed to be in slight interaction. Without interaction, either direct by collisions or indirect through a common heat bath, one never would arrive at a thermal equilibrium. On the other hand, if the interaction were not *slight* then it would be meaningless to speak of the various parts of the macrosystem as individuals; a clear division into parts is possible only in the limit of negligibly small mutual effects. Therefore, when considering bodies whose parts are in strong interaction, statistical thermodynamics can be applied only to a Gibbsian ensemble of many samples of such a body. Statistical-thermodynamic phenomena are dependent, however, on the resonance mode of interaction between the slightly interacting parts of the assemblage.

(a) Resonance Interaction due to Superposition Mechanics

Let us start with a system of only *two* identical particles 1 and 2, whose states a and b may be described by amplitudes ψ_a and ψ_b as eigenfunctions of an observable $H^0(1,2) = H^0(1) + H^0(2)$. The superscript⁰ is to indicate lack of mutual perturbation between 1 and 2. An eigenvalue of H^0 is $E_a^0 + E_b^0$; it belongs to the two ground solutions $\psi_a(1)\psi_b(2)$ and $\psi_a(2)\psi_b(1)$ as well as to any of their linear combinations. As soon as H^0 is augmented by an ever so small term $H'(1,2)$ symmetric in 1 and 2 (because 1 and 2 are identical particles) then even in zero approximation only the symmetric and the antisymmetric combination of the two ground solutions is admitted:

$$\psi^0(1,2) = \psi_a(1)\psi_b(2) \pm \psi_a(2)\psi_b(1). \quad (3)$$

This *resonance effect* is usually derived as a result of the h -controlled Schrödinger wave mechanics in the Heisenberg and Heitler-London theory. Yet the same result is inherent already in the general superposition formalism without reference to any special connection between conjugates q and p . The same applies to the further result that the eigenvalue $E_a^0 + E_b^0$ is to be augmented by a Coulomb-like interaction term C_{ab} and by an exchange or resonance term D_{ab} with positive or negative sign in the symmetric or antisymmetric case respectively. The eigenvalue of $H^0 + H'$ thus becomes

$$E = (E_a^0 + E_b^0) + C_{ab} \pm D_{ab}. \quad (4)$$

The same result is also described by the following scheme:

ψ	$a_1 b_2$	$a_2 b_1$	
sym	1	1	$+D_{ab}$
ant	1	-1	$-D_{ab}$

where $a_1 b_2$ in the top row stands for $\psi_a(1)\psi_b(2)$; the next two rows give the factors in the symmetric and antisymmetric combination. The last column lists the exchange term. For later use we add the corresponding scheme resulting from the resonance theory of three particles, 1 and 2 and 3 in three states, a and b and c . The factors supplied to the $3! = 6$ ground solutions are powers of the sixth root of unity,

$$f = \exp(2i\pi/6), \text{ hence } f^n = \exp(2i\pi n/6).$$

ψ	$a_1 b_2 c_3$	$a_1 b_3 c_2$	$a_2 b_3 c_1$	$a_3 b_1 c_2$	$a_3 b_2 c_1$	
...	f	f^2	f^3	f^4	f^5	f^6
...	f^2	f^4	f^6	f^8	f^{10}	f^{12}
ant	f^3	f^6	f^9	f^{12}	f^{15}	f^{18}
...	f^4	f^8	f^{12}	f^{16}	f^{20}	f^{24}
...	f^5	f^{10}	f^{15}	f^{20}	f^{25}	f^{30}
sym	f^6	f^{12}	f^{18}	f^{24}	f^{30}	f^{36}

One may think of the example of three identical electrons in the fields of three different centers, a , b , and c . There are then six different ways of reaction between the three particles, among them the symmetric one listed in the last row where all factors f^n are unity, and the antisymmetric one in the third row with factors -1 and $+1$, alternately. The other rows belong to other types of resonance reaction shortly called "unsymmetric"; their exchange energies are involved expressions. The transition from the first to the second scheme may be exemplified by the addition of a third atom c to a molecule ab so as to

form a triple molecule. Perturbation theory of superposition mechanics admits all six modes of resonance reaction, although there is no guarantee that any of them will lead to a stable triple molecule.

The following two questions now arise: Why is it that in case of two, and three, and N identical particles only *one* out of $2!$, $3!$, and $N!$ symmetry types of reaction is actually present? And further: Why is it that the symmetry type once chosen by two particles is conserved also when a third, a fourth, and an N th particle of the same kind is added to the system? Both questions are answered by pointing to the principle of continuity. Let us first consider the reduction from $N!$ to *one*.

(b) Reduction from $N!$ Modes of Interaction to One

This reduction is necessary in order not to come in conflict with the Gibbs discontinuity paradox of classical statistical thermodynamics. Classical theory, in contrast to observation, yields a finite entropy increase during the diffusion of two identical samples of a gas. Indeed, according to classical statistics, the entropy of N particles at temperature T in a volume V is described by the well-known formula

$$S = kN \{ \lg V + C \lg T + \text{const} \}, \quad (5)$$

where k is the Boltzmann constant, C is a specific heat constant, ($3/2$ for monatomic, and $5/2$ for diatomic gases), and const is independent of V , T , and N . However, this classical S formula leads to the unacceptable result that the diffusion of, say, 5 like gas samples from their original separate volumes V into the common volume $5V$ would lead to an entropy increase. Indeed, for 5 separate samples one has, according to Eq. (5), the total entropy

$$S_0 = 5kN \{ \lg V + C \lg T + \text{const} \}.$$

After their diffusion the gas of $5N$ particles in $5V$ at the same T would be

$$S = k5N \{ \lg(5V) + C \lg T + \text{const} \},$$

leading to the entropy increase

$$S - S_0 = 5kN \lg 5 \quad (\text{classical}), \quad (5')$$

instead of $S - S_0 = 0$. The classical entropy formula is wrong. In order to arrive at a correct expression for the entropy of a gas of N particles in V at T we may tentatively replace the *const*, which certainly must be independent of V and T , by a still unknown function of N ; that is, let us tentatively replace Eq. (5) by

$$S = kN \{ \lg V + C \lg T + f(N) \}, \quad (6)$$

and determine $f(N)$ so that $S - S_0$ for the diffusion of 5 like gases will vanish. Originally we have, according to Eq. (6),

$$S_0 = 5kN \{ \lg V + C \lg T + f(N) \}.$$

After merging the $5N$ particles in $5V$ we obtain

$$S = k5N \{ \lg(5V) + C \lg T + f(5N) \}.$$

In order that $S - S_0 = 0$ we must have

$$f(5N) - f(N) = -\lg 5 = -\lg 5 + \lg 1,$$

from which we learn that $f(N)$ must equal $-\lg N + \text{const}$. The correct entropy formula for N particles in V at T thus reads

$$S = kN \{ \lg V + C \lg T - \lg N + \text{const} \}. \quad (7)$$

Equation (7) is obtained from Eq. (5) by subtraction of the term $kN \lg N$. Now, since S stands for Boltzmann's $k \lg P$, and since $N \lg N$ is only the Stirling approximation of $\lg N!$ in statistical calculations, the subtraction of $kN \lg N \sim k \lg N!$ from the classical entropy S signifies a *division* of the classical probability P by $N!$. Hence, whereas classical statistics counts $N!$ micro-distributions of N particles over N energy levels E_a, E_b, \dots, E_n belonging to one and the same macro-configuration of energy $E_a + E_b + \dots + E_n$, the correct statistics has to count such a macro-configuration with statistical weight *unity* rather than $N!$. In superposition mechanics the same $N!$ permutations yield $N!$ resonance modes of interaction; *only one of them can actually be admitted* for the sake of (7) replacing (5).

(c) Conservation of Symmetry Type

All unsymmetric functions ψ must immediately be discarded as unphysical since they have, as can be seen easily, *unsymmetric probabilities* $|\psi|^2$, unfit to represent legitimate states of identical particles. This leaves for any number N

the choice between symmetry and antisymmetry only.

Considering now a certain kind of identical particles, there still might be the possibility that these particles in two's may choose the symmetric way of resonance interaction listed in the first line of the 2-particle scheme, but might, in case of three's, choose the antisymmetric resonance interaction of the 3-particle scheme. That this cannot be so, and that there rather must be conservation of symmetry type when going from 2 to 3 to N identical particles can be proved as follows: Suppose that the two particles under consideration interact the symmetric way with exchange energy $+D_{ab}$ in the molecule ab (to use a familiar example). Suppose there is a third particle 3 in the state c , without any interaction with particles 1 and 2, e.g., bound in an atom c infinitely removed. The total resonance energy of the system $ab+c$ will still be $+D_{ab}$ since there is no mutual energy between a and c nor between b and c . When we now move the atom c from infinity to an ever-so-large *finite* distance from the molecule ab to form what may be called a triple molecule abc , then the absolute values of the exchange terms D_{ac} and D_{bc} will still be negligibly small in magnitude. The total exchange energy of the system abc of the three particles in antisymmetric resonance would be $-D_{ab}$. There would then be an abrupt change of the original exchange energy $+D_{ab}$ of the system upon the approach of c from an infinite to an ever-so-large distance from ab , except in the case of symmetric interaction also in case of the 3-particle system. Thus, in order to safeguard *continuity of the resonance energy* upon the incorporation of one more particle into a 2-particle system, and similarly during the addition of any new particle, there must be the rule *once symmetric always symmetric*, and similarly *once antisymmetric always antisymmetric*. The rule is identical with the division of all particles in two classes, symmetrically interacting Bose particles, and antisymmetrically interacting Fermi particles. The latter, of course, obey the Pauli exclusion principle, since antisymmetric ψ functions for states $a, b, c \dots n$ vanish when two or more of these states are the same.

The statistical weight *unity* due to the absence of a $N!$ -fold degeneracy leads immediately to the

consequence that the average number of particles per energy level ϵ at temperature T is

$$\bar{n} = \left(\frac{1}{\xi} e^{\epsilon/kT} \mp 1 \right)^{-1} \quad (8)$$

with upper sign for Bose and lower sign for Fermi particles. The rule of symmetry-type conservation remains in force under any continuous variation of the *magnitude* of the interacting forces involved. When it is true for one state of aggregation under small forces (in a gas state), it remains also true when the forces are made large (in a liquid or solid state). And since nondegeneracy rather than $N!$ fold degeneracy also prevails for the very lowest energy level in any state of aggregation, the precondition for the *Nernst theorem* is satisfied.

All unsymmetric ψ 's can be shown to belong to at least twofold degenerate energy levels. The Nernst theorem and the requirement that $|\psi|^2$ be symmetric are thus equivalent: Both eliminate the unsymmetric ψ functions.

3. CANONICAL CONJUGACY AND QUANTUM DYNAMICS

Classical dynamics rests on the Hamiltonian equations of motion for coordinates q and momenta p , defined as being conjugate when they obey the canonical equations. Quantum mechanics usually introduces conjugacy either by the Born commutation rule $qp - pq = i\hbar$, or by the Schrödinger operator $p = (\hbar/i)\partial/\partial q$, both definitions being equivalent. Instead, we wish to introduce the following general *qualitative* definition: "In every mechanical system there are pairs of observables q and p so that the manifold of all q observations *uniquely* determines the manifold of all p observations." For example, when the energy E and the time t are said to be conjugate this shall imply that an observable f , whose eigenvalues are $f_u = f(t)$, whereas $f_{uv} = 0$ for $t \neq t'$, has transition values $f_{E'E''}$ which are nonvanishing only for certain energy transitions $\Delta E = E' - E''$ *uniquely* determined by $f_{uv} = f(t)\delta_{uv}$. A consequence of this qualitative definition of conjugacy between E and t is the quantitative result that $\psi(E, t)$ has

the form $\exp(iEt/\hbar)$, where \hbar is a constant of the dimension of the product Et , whose magnitude can of course not be predicted in advance. The proof runs as follows.

According to the general transformation rule (2) of superposition one has, replacing summation by integration from $t = -\infty$ to $+\infty$,

$$f_{E'E''} = \int \psi(E', t) f(t) \psi(t, E'') dt, \quad (9)$$

where $\psi(t, E'') = \bar{\psi}(E'', t)$. Now, when the left-hand side is to have nonvanishing values only for certain energy differences $\Delta E = E' - E''$, then these differences must occur on the right-hand side in the product $\psi(E', t)\bar{\psi}(E'', t)$, since $f(t)$ does not contain E . This occurrence of the *difference* is possible only when the individual product factors ψ and $\bar{\psi}$ are *exponential* functions containing E as linear terms in the exponent, multiplied in ψ by i and in $\bar{\psi}$ by $-i$ in order that E' and E'' appear in ψ and $\bar{\psi}$ with opposite signs. That is, $\psi(E, t)$ must be of the form

$$\psi(E, t) = \exp(\dots iE \dots), \quad (10)$$

irrespective of the special function $f(t)$, i.e., of the special mechanical system under consideration. Now, since conjugacy is mutual, one may carry through the same argument with the rôle of E and t reversed. That is, assuming that an observable g whose matrix elements are $g_{EE'} = g(E)\delta_{EE'}$, uniquely determines certain intervals $\Delta t = t' - t''$ for which $g_{t't''}$ does not vanish, one arrives at the conclusion that $\psi(E, t)$ must also have the form

$$\psi(E, t) = \exp(\dots it \dots), \quad (10')$$

irrespective of any particular system under consideration. Comparing Eq. (10) with Eq. (10') one arrives at the following alternative. Either $\psi(E, t)$ contains iE and it separately in the form $\exp(\dots ait \dots biE \dots)$, or the same factor i belongs to both E and t simultaneously in the form

$$\psi(E, t) = \exp(\dots iEt \dots) = A \exp(iEt/\hbar), \quad (11)$$

where \hbar is a constant of the dimension of an action. Only the latter alternative is admissible. Indeed, when considering $f(t)$ in the harmonically

* E. Schrödinger, *Statistical Thermodynamics* (Cambridge University Press, Cambridge, 1946), Chapter III.

expanded form

$$f(t) = \sum_k f_k \cos(\omega_k t) = \sum_k \frac{1}{2} f_k (e^{i\omega_k t} + e^{-i\omega_k t}) \quad (12)$$

and substituting the first alternative for $\psi(E, t)$ into Eq. (9) then $+ait$ and $-ait$ cancel in the product $\psi \bar{\psi}$, leaving

$$f_{E'E''} = \sum_k \frac{1}{2} f_k \exp(\dots bi\Delta E \dots) \int \exp(\pm i\omega_k t) dt.$$

The integrals *vanish* essentially, i.e., they fluctuate between positive and negative values depending on exactly where one stops integrating toward $t = \pm \infty$. With the second alternative (11), however, one obtains

$$f_{E'E''} = \sum_k \frac{1}{2} f_k \int \exp[i(\Delta E/\hbar \pm \omega_k)t] dt, \quad (13)$$

which has nonvanishing (and finite, if normalized properly) values for those ΔE values which satisfy the condition

$$|\Delta E| = \omega_k \hbar, \quad (14)$$

determined uniquely by the function $f(t)$, in particular by the harmonic frequencies ω_k . Only the alternative (11) can be correct.

We have thus arrived at the *general* result that the probability amplitude $\psi(E, t)$ must have the complex exponential form (11) periodic with frequency $\omega = E/\hbar$. This result is identical with Schrödinger's replacement of the energy E by the differential operator $(\hbar/i)\partial/\partial t$, and when q and p are conjugate in the same sense, of the momentum p by $(\hbar/i)\partial/\partial q$. The latter result is

equivalent with the Born commutation rule, $pq - qp = i\hbar$.

We have further obtained in Eq. (14) the *Bohr frequency condition*: A system with a spectrum of frequencies ω_k must have energy level differences $\Delta E = E' - E'' = \omega_k \hbar$.

The counterpart of an oscillator in time is a periodic crystal in space, described by the vector function

$$f(\mathbf{r}) = \sum_{klm} f_{klm} \cos(k\tilde{\omega}_1 \mathbf{r}_1 + l\tilde{\omega}_2 \mathbf{r}_2 + m\tilde{\omega}_3 \mathbf{r}_3), \quad (15)$$

from which the Laue-Bragg theory derives maxima of interference in certain selected directions. The same maxima can be obtained also from the Ewald reciprocal lattice construction which, after multiplication by h is a lattice construction in momentum space³ and tells us that the crystal described in space by the values $f_{rr'} = f(\mathbf{r})\delta_{rr'}$ of Eq. (16) is a system capable of giving out certain selected vector momentum amounts $\Delta \mathbf{p} = \mathbf{p}' - \mathbf{p}''$ uniquely determined by the function $f(\mathbf{r})$.

It is gratifying that the quantitative rules of Planck ($\Delta E = h\nu$ for an oscillator), Bohr ($\nu = (E' - E'')/h$), Schrödinger, and Max Born do not have to be *assumed* but may be *derived* from the above qualitative definition of conjugacy interpreted in terms of the general superposition formalism which, in the last resort, springs from the postulate of thermodynamic continuity.

³ Refer to A. Landé, *Quantum Mechanics* (Pitman Publishing Corporation, New York, 1951), §5, p. 20.

Southern California Section

The Southern California Regional Section of the American Association of Physics Teachers announces its Tenth Annual High School Physics Test, to be given Saturday, May 1, 1954, at Compton College, Santa Barbara, University of Redlands, and elsewhere as in other years. Details may be secured from PROFESSOR COLLIS M. BARDIN, Chairman, Test Committee, Compton College, Compton, California.

JULIUS SUMNER MILLER, *Secretary*

Installation and Operation of Ten-Kilocurie Cobalt-60 Gamma-Radiation Source*

JOHN V. NEHEMIAS, L. E. BROWNELL, W. W. MEINKE, AND E. W. COLEMAN
University of Michigan, Ann Arbor, Michigan

(Received March 13, 1953)

The expanding interest in studies of the effects of gamma radiation upon biological and chemical systems has encouraged the production of increasingly higher level sources for various institutions. Recently, at the University of Michigan, a ten-kilocurie cobalt-60 gamma radiation source has been installed and is now in routine operation. A brief discussion of some of the problems involved in installation and operation of such a source and some of the studies being undertaken is presented.

SINCE early in 1951, the Fission Products Laboratory of the Engineering Research Institute of the University of Michigan has been investigating possible industrial uses for the highly radioactive fission products being produced as waste by normal reactor operation.¹⁻³ This waste material, because of its metabolic and radiological characteristics, cannot be disposed of by ordinary methods, but must be permanently stored. If large-scale uses can be found for the radiation from fission products, an expensive, hard-to-handle waste product can become valuable as an industrial tool.

Since high-level radiation sources prepared from fission products have not been available, pilot-type studies are proceeding using pile-produced isotopes, primarily cobalt-60.⁴ The original source in use at this laboratory is a hollow cylinder of metallic cobalt, containing about one kilocurie of cobalt-60. Figure 1 illustrates the container used for this source. The lead plug is removed by a lift and sample tubes about one and one-half inches by ten inches are lowered into place inside the cylinder of cobalt, where the dosage rate is of the order of 75 000 rep/hr.

* This work was supported in part by the U. S. Atomic Energy Commission and in part by Michigan Memorial Phoenix Project.

¹ L. E. Brownell *et al.*, *Utilization of the Gross Fission Products*, Progress Report 1, Engineering Research Institute, Project M943, University of Michigan, U. S. Atomic Energy Commission Contract No. AT(11-1)-162 (1952). (C00-86, August 1952).

² L. E. Brownell *et al.*, Progress Report 2, (C00-90, January 1952).

³ L. E. Brownell *et al.*, Progress Report 3, (C00-91, June 1952).

⁴ *Fission Products Utilization*, Brookhaven National Laboratory Conference Report, BNL 171 (C-14), February 1952.

In order to increase the radiation intensity available and permit the irradiation of greater volumes of material, it was decided that a new source, ten times as powerful, and with a somewhat different geometrical arrangement, should be acquired. Figure 2 is a phantom view of the new installation, approximately as it is now in use. With this source, the entire room is used as a radiation "cave." Dosage rates vary from

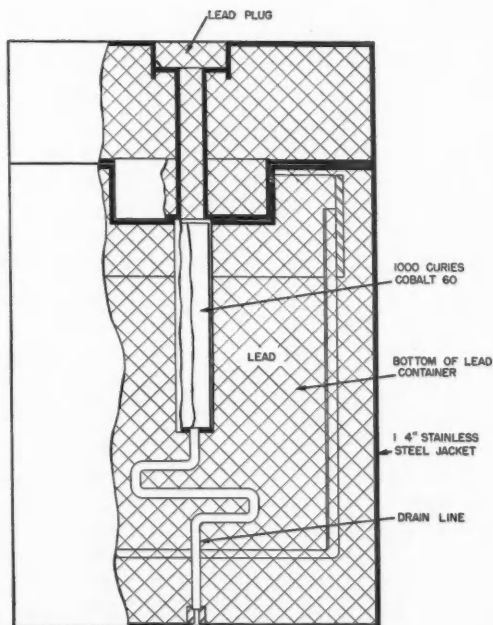


FIG. 1. Section view of 1000-curie cobalt-60 source. Samples to be irradiated are lowered remotely, with the lead plug removed, into the hollow cobalt cylinder. Dosage rate there is about 75 000 rep/hour.

around 2,000 rep/hr at the farthest point in the cave to around 300 000 rep/hr inside the source.

OPERATION

The source consists of one hundred rods of metallic cobalt, jacketed with aluminum to prevent corrosion. These rods are arrayed in a hollow cylinder, about nine inches in diameter, and held in place by a plastic rack, shown in the operating position with no radioactive material in Fig. 3. This rack for holding the source rods rests on an elevator platform, shown in Fig. 4,

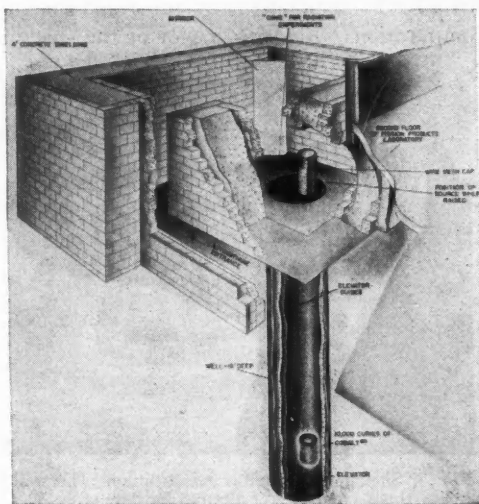


FIG. 2. Phantom view of 10 000-curie cobalt-60 source. During operation the source is up inside the Lucite cap. Dosage rate within the source cylinder is about 300 000 rep/hr. With the source down in the well, it is safe to enter the cave to arrange irradiation experiments.

which is raised and lowered by a cable operated from outside the cave to permit sample changing and positioning. With the source at the lower position, the operator can spend several hours in the cave, if necessary, without exceeding the tolerance dose.

Material to be irradiated at the high-flux position in the center of the source may be packaged in containers as small as a test tube or as large as a No. 10 can. These containers are then placed within the central cylinder of the Lucite cap, shown in Fig. 3. When the source is raised it fits into the Lucite cap, which is mounted at the top

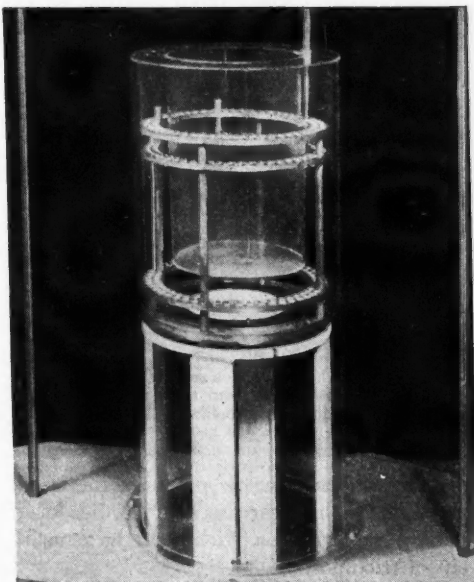


FIG. 3. Dummy source assembly in operation position. The source assembly consists of 100 aluminum-jacketed cobalt rods, held in place by a plastic rack resting on a steel platform. How the source surrounds the central high-flux irradiation position is clearly shown.

of the well. Samples placed around the outside of the Lucite cap are slowly rotated with small turntables to provide more uniform irradiation of the material. If uniformity of irradiation is not of primary importance, much larger samples may

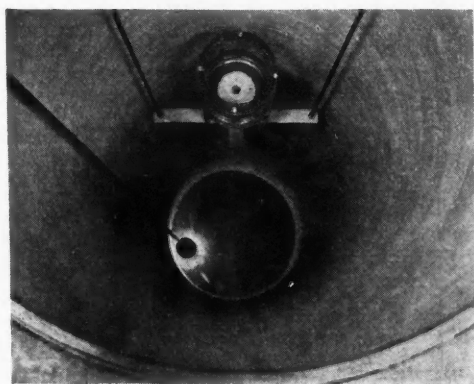


FIG. 4. Elevator assembly part-way into empty well. The well is lined with a water-cured ceramic to reduce corrosion. The source is acentrically located in the well to facilitate loading the source rods in the rack.

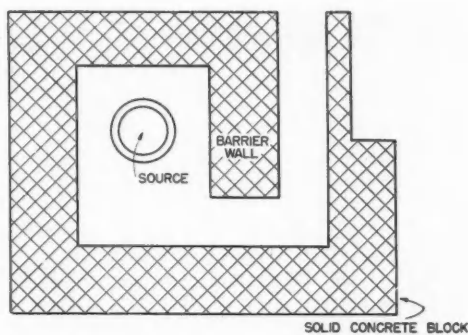


FIG. 5. Floor plan of radiation cave. The drawing shows the radiation installation as it is related to the laboratory. Unloading the source rods from their shipping container into the well was done from behind the barrier wall with the use of mirrors.

be treated. (One group is planning the irradiation of entire hog carcasses in studies of the effects of radiation on trichinae, the causative agent of trichinosis).

When the samples are arranged according to desired dosage rate, the operator leaves the cave

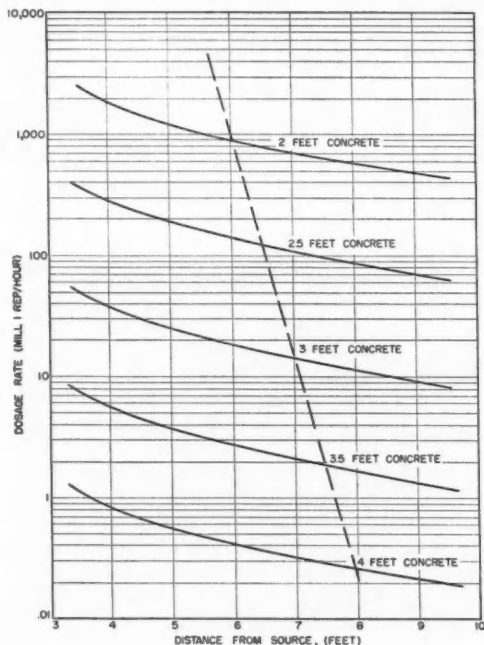


FIG. 6. Dosage rate vs concrete thickness. The dotted line represents the position and dosage rate of the outer surface of a shield four feet from the source. A shield thickness of four feet was chosen for safety and convenience.

and bolts it shut. He then steps to the control winch and raises the source into its operating position.

SHIELDING

The dimensions selected as optimum for the radiation cave were 8 ft \times 11 ft \times 8 ft. This places the inner surface of the shield wall four feet from the center of the source (see Fig. 5). With the shield so situated, it was necessary to determine the thickness of concrete required to reduce radiation levels at the outer surface to safely below tolerance values.

Based on predicted dosage rates⁵ and shielding computations,⁶ Fig. 6 is a plot of the shielding

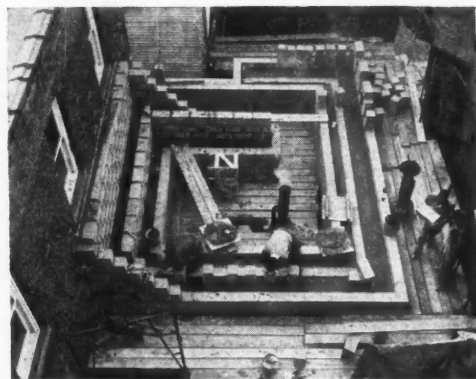


FIG. 7. Shielding wall under construction. The wall is four feet thick, composed of alternating rows of 8 \times 8 \times 16-in. solids blocks and poured concrete, as shown.

effects of various thicknesses of concrete as a function of position. The dotted line indicates dosage rate outside a concrete shield whose inner face is four feet from the source, as a function of thickness.

As the outer surface is accessible to the general public a maximum design dosage rate of one mr/hr was considered permissible. Since it is convenient to lay concrete walls in units of eight inch thickness, and the associated factor of safety is not exorbitant, a thickness of four feet of concrete was selected. The wall is shown in Fig. 7 in the process of construction. Dosage rate at the outer surface of the shield wall is less than one mr/hr.

⁵ G. W. Morgan, U. S. Atomic Energy Commission Isotopes Division Circular, B-3, 1948.

In order that it be safe for workers to enter the cave for extended periods, it is of importance to determine similarly the required thickness of water shielding. On the basis of the same considerations, a shielding thickness of eleven feet was considered sufficient. However, in addition to shielding thickness, five feet of depth are required for manipulation at the top and bottom of the well, and by the design of the raising mechanism. Therefore, a sixteen-foot well was prepared.

To facilitate the installation procedures, described in a later section, no overhead shielding

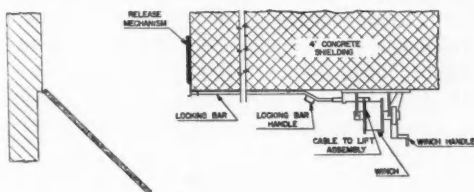


FIG. 8(A). Mechanical safety interlocks. When the cave door is open, the spring release prevents the locking bar from sliding to the left and freeing the winch for raising the source. With the door open, the source cannot be raised.

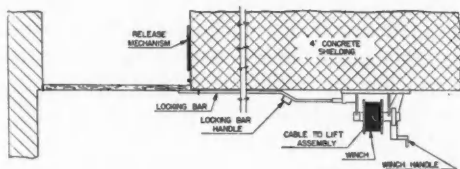


FIG. 8(B). Mechanical safety interlocks. When the source is up, the locking bar prevents the door from being opened. The locking bar is prevented from sliding to the right by only one turn of the cable on the winch. With the source up, the door cannot be opened.

was incorporated in the original structure. When the installation was complete, however, it was found that the secondary radiation from the cave walls and roof was sufficient to raise radiation levels in nearby offices and dormitories above tolerable values. Concrete was then added overhead, up to two feet thick directly over the source and extending at lesser thicknesses over the passageway beyond the barrier wall. This final shielding reduces the neighborhood radiation levels to essentially negligible values.

SAFETY PRECAUTIONS

Because it is possible to receive about five hundred rep (generally considered lethal) in six

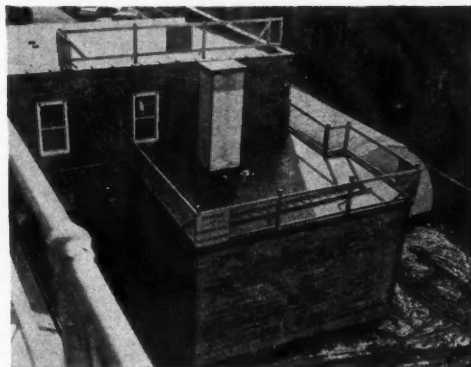


FIG. 9. External view of radiation cave. With the source in the operating position the roof of the cave is an extremely dangerous area, and is clearly so marked. Radiation levels in the laboratories on the left, the parking lot to the rear and the dormitories on the right, however, are well below tolerance values.

minutes at the edge of the room, and in even less time toward the center, much emphasis has been given to safety considerations.

A mechanical interlock, shown in Fig. 8(A), prevents the source from being raised when the entrance to the cave is open. It is possible for a person entering the cave to padlock the source in the down position, and thus prevent it from being raised while he is in the cave.

A second position of the interlock, Fig. 8(B), will prevent the entrance to the cave from being opened when the source is in the up position. In the event that the elevator is lowered normally, but the source remains stuck in the Lucite cap, as a result of any possible misalignment, a weight-operated red warning signal will indicate that the source is still in the up position. In addition to these precautions no one enters the cave without first determining visually, through a double mirror system, and with portable radiation monitoring instruments, that the source is indeed down.

The roof of the cave building, with only two feet of concrete, is a very dangerous area. It is clearly marked and fenced off with barbed wire as shown in Fig. 9.

INSTALLATION

The source rods arrived in a three-ton lead shipping container about two feet in diameter

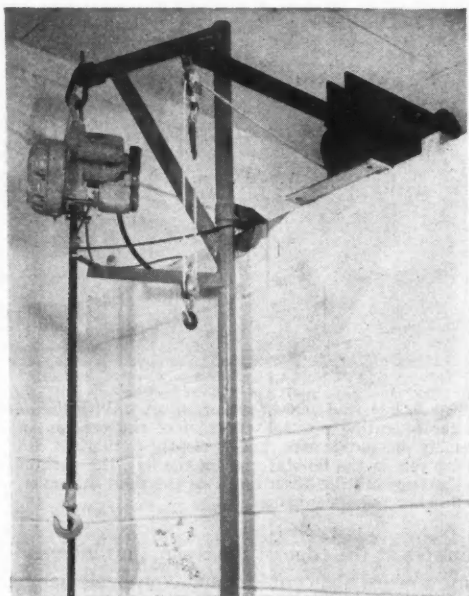


FIG. 10. Unloading apparatus for source transfer. In transferring the source rods from the shipping container to the well, the heavy chain hoist was used to remove the cover (about 300 pounds) and the rope and pulley to raise the source material, roll it over the well and lower it into the water. The operator was behind the barrier wall.

and three feet high. The container was set into place between the well and the barrier wall.

The lifting apparatus, shown in Fig. 10, was connected to the lid of the shipping container while the container was still closed. The lid, weighing about three hundred pounds, was raised and set out of the way by personnel operating from behind the barrier wall. The bucket of source rods was then fastened remotely to the

second lifting rig, lifted from the shipping container, and lowered into the well. The source was exposed in air during transfer for less than 30 seconds.

APPLICATION

In the experimental studies the effects of gamma radiation on biological systems and on chemical reactions are being investigated. Gamma radiation has been found lethal to all organisms studied to date, providing sufficient dosages are used. Making use of this characteristic of radiation, pharmaceutical products have been sterilized, foods have been preserved and the cycle of diseases such as trichinosis has been broken. Gamma radiation has also been used to promote chemical reactions of industrial importance such as chlorination and polymerization.¹

SUMMARY

Several one-to-five-kilocurie sources of gamma radiation are now in use in the United States. Most of these have an irradiation volume similar to the one kilocurie source in use at this laboratory. The source recently installed at this laboratory, however, contains ten kilocuries of cobalt-60 and has an inside irradiation volume about nine inches in diameter and ten inches long. In addition to this high-flux volume, the entire inside of the cave can be used for lower flux irradiation. With this source, it is possible to produce the relatively large quantities of irradiated materials required for pilot studies of possible biological uses for high-level gamma radiation and provide the flexibility required for chemical studies.

Central Pennsylvania Section

The Central Pennsylvania Section of the American Association of Physics Teachers will meet at *Juniata College*, Huntington, Pennsylvania, on April 9 and 10, 1954. Local chairman for the meeting is PAUL R. YODER, *Juniata College*.

DOROTHY W. WEEKS, *Secretary*

NOTES AND DISCUSSION

Application of Matrix Algebra to Electric Networks

R. A. HOWARD

University of Oklahoma, Norman, Oklahoma

IN a recent book¹ on the application of matrix algebra to linear electrical networks some very interesting results originally published by Kron² are derived by introducing auxiliary networks. A simpler and more satisfying derivation can be carried out without the introduction of these auxiliary networks.

Consider a circuit made up of any number of branches, each branch containing a source of emf, and having a self-impedance Z_{ii} and mutual impedances Z_{ij} with other branches of the network. The sources of emf may be ac generators provided the frequencies are the same. A branch current will be considered positive if it flows in the direction which a current would flow if the network consisted only of resistors and all emf's were zero except the one in the branch under consideration. The potential difference between the terminals of the branch will be considered positive if the assumed branch current flows from the lower potential to the higher potential terminal of the branch.

One may then write the following matrix equation for the various branches:

$$V = e - Zi, \quad (1)$$

where the matrix elements are generally complex quantities, in keeping with the usual method of treatment of alternating current circuits.

If one now considers the circuit from the standpoint of its meshes, one can express the branch currents in terms of the mesh currents which flow through the individual branches by the matrix relation

$$i = Ci', \quad (2)$$

where the elements of i' are the mesh currents. Following LeCorbeiller, the instantaneous power dissipation in the circuit must be the same whether calculated in terms of the branch or the mesh quantities. Thus

$$P = \frac{1}{2}(e_i^* i + e_i i^*) = \frac{1}{2}(e_i'^* i' + e_i' i'^*),$$

where the primes refer to mesh quantities, stars represent complex conjugates, and t indicates the transpose of the matrix in question. If one substitutes for i from Eq. (2), he obtains

$$(e_i'^* - e_i^* C) i' + (e_i' - e_i C) i'^* = 0,$$

which, since C is real, can be true for all i' only if

$$e_i' = e_i C,$$

or, taking the transpose of both sides of the equation,

$$e' = C e. \quad (3)$$

If Eq. (1) is now multiplied through by C_i and use is made of Eqs. (2) and (3),

$$C_i V = C_i e - C_i Z i = e' - C_i Z C i'.$$

Physically, a mesh emf is simply the sum of the branch emf's in the mesh considered, so that C_i is an operator which adds up the emf's in a mesh. Since the potential differences are taken with the same sign convention as the emf's, $C_i V$ must represent the sums of potential differences around the meshes. But Kirchhoff's second law says that $C_i V = 0$, so that

$$e' = C_i Z C i'.$$

If one were to start with the mesh equations, he would write

$$e' = Z' i',$$

so that evidently

$$Z' = C_i Z C. \quad (4)$$

Equation (4) is the important relation between branch and mesh impedances which it was the purpose of this note to show could be derived without the introduction of auxiliary circuits.

¹ P. LeCorbeiller, *Matrix Analysis of Electric Networks* (Harvard University Press, Cambridge, 1950).

² Gabriel Kron, *Tensor Analysis of Networks* (John Wiley and Sons, New York, 1939).

Equivalence of the Clausius and Kelvin Postulates

CHARLES N. HAMTIL

Rockhurst College, Kansas City, Missouri

IT is customary in introducing the second law of thermodynamics to cite two principles drawn from experience; one of these is due to Kelvin, the other to Clausius. The Kelvin postulate is stated as follows: it is impossible to construct a device which, working in a cycle, can extract heat from a reservoir and perform an equivalent amount of work without producing any other effects. The Clausius postulate is as follows: it is impossible to construct a device which, working in a cycle, can effect the transfer of heat from a cooler to a hotter body without producing any other effects.

In most treatises on thermodynamics, physical arguments are employed to show in a general way that an assumed violation of the Clausius postulate, for instance, leads to a violation of the Kelvin principle; but seldom is the relationship between these two statements examined fully enough to establish their logical connection in a comprehensive way.

Among the authors that treat this question with some care is Zemansky,¹ who (a) shows, by physical reasoning, that falsity of the Clausius statement implies falsity of the Kelvin statement; (b) proves, also by physical arguments, that falsity of the Kelvin statement implies falsity of the Clausius; and (c) sets up the symbolic machinery for further logical analysis. It is concluded that the Clausius and Kelvin postulates are entirely equivalent statements of the second law, but the details of the symbolic demonstration are omitted.

What follows in this paper is an elementary symbolic proof of the logical equivalence of the Clausius and Kelvin postulates. Such elements of symbolic logic as are needed in the proof are incorporated into the presentation² so that no knowledge of modern logic need be presumed on the part of the student. It should be possible to give the entire proof within the compass of a single class period to students who have just been introduced to the second law of thermodynamics.

Let any proposition be denoted by the letter p . The opposite or negation of p will be denoted by the symbol $\sim p$ (read: not p).

When a proposition p implies³ another proposition q , we shall denote this relationship symbolically by writing

$$p \rightarrow q.$$

When a proposition p implies a proposition q , and q in turn implies p , we shall write

$$p \leftrightarrow q.$$

Whenever p and q are connected by this relation of mutual implication, they are termed *equivalent* propositions.

Let us suppose the premise

$$p \rightarrow q.$$

Now assume that q is a false proposition. It follows that p must also be false; for, if p were true, q would also be true by the very meaning of implication. Thus, falsity of q implies falsity of p . This result may be concisely expressed in symbolic notation as follows: given the premise

$$p \rightarrow q,$$

it follows that

$$\sim q \rightarrow \sim p.$$

This theorem is called the *opposite converse rule of inference*.

Let K and C denote the Kelvin and Clausius postulates, respectively. The physical considerations introduced above have led to results that may be summarized in symbolic notation by writing

$$\sim K \rightarrow \sim C, \quad (1)$$

$$\sim C \rightarrow \sim K. \quad (2)$$

Our task is then as follows: given that relationships (1) and (2) are true propositions, we have to prove that

$$K \leftrightarrow C,$$

i.e., that K and C are equivalent.

We begin with relation (1).

$$\sim K \rightarrow \sim C.$$

This expression may be transformed by the opposite converse rule of inference into

$$\sim \sim C \rightarrow \sim \sim K,$$

which, by reason of the double negative, immediately becomes

$$C \rightarrow K. \quad (3)$$

Similarly relation (2),

$$\sim C \rightarrow \sim K,$$

becomes, upon application of the opposite converse rule,

$$\sim \sim K \rightarrow \sim \sim C,$$

which is equivalent to

$$K \rightarrow C. \quad (4)$$

Relations (3) and (4) may now be combined into the single expression

$$K \leftrightarrow C,$$

which affirms the equivalence of the Clausius and Kelvin postulates.

¹ Zemansky, *Heat and Thermodynamics* (McGraw-Hill Book Company, Inc., New York, 1951), pp. 146-150.

² A very readable introduction to symbolic logic is given by E. R. Stabler, *Introduction to Mathematical Thought* (Addison-Wesley, Publishing Company, Inc., Cambridge, 1953), Chap. III. The ideas introduced in the next two sections of this paper are based largely on Stabler's presentation.

³ The meaning of the word *implies* is, for our purposes, sufficiently evident from logical intuition. For a formal symbolic definition of implication see Stabler, reference 2.

Concerning the Frequencies Resulting from Distortion

F. W. VAN NAME, JR.

Franklin and Marshall College, Lancaster, Pennsylvania

RECENTLY Robinson has suggested¹ that since harmonic analyzers are known to work, the sinusoidal components of a Fourier analysis of an arbitrary wave form must be "physically real and present in the system."

This writer would not deny that harmonic analyzers do indeed perform a Fourier analysis experimentally. My objection is to the conclusion that such an analysis is *unique*, giving the resulting components the status of reality. Since the response of many circuits to sine waves is well known, the Fourier analysis of a complex wave is often convenient, but it is possible to resolve a wave in terms of alternative sets of orthogonal functions.

In a different connection Van Der Pol shows² that this can be done for a saw-tooth wave. He illustrates the synthesis of a saw-tooth wave first with a set of orthogonal square waves of increasing harmonic frequencies and then with the usual Fourier series. Thus, if a saw-tooth wave is to be resolved into components, the resolution can be carried out in terms either of a series of square waves or in terms of a series of sine waves. In view of this possible choice, one could hardly say that the particular set chosen was physically real. This writer believes that only the actual wave form can be said to be physically present.

¹ Berol L. Robinson, *Am. J. Phys.* **21**, 391 (1953).

² Balh. Van Der Pol, *J. Franklin Inst.* **255**, 475 (1953).

LETTERS TO THE EDITOR

Linear Relativistic Hamiltonians

IN a recent letter to the editor,¹ Henri Mitler gives a method of linearizing the relativistic Hamiltonian for a free particle, using three anticommuting two-rowed matrices instead of the usual set of four anticommuting four-rowed matrices. Since it is known to be impossible to carry out the linearization satisfactorily with two-rowed matrices, Mr. Mitler concludes that there must be some mathematical error in his work.

I do not believe that there is any error in his work, as far as it goes. However, any attempt to use his Eq. (9) or Eq. (11) as a basis for a physical theory would run into serious difficulty, because these equations are not in relativistically proper form. In fact, no linear energy-momentum equation that uses only three anticommuting matrices can be relativistically correct because, in a four-dimensional manifold, three matrices are not enough to go round. One way of explaining this is as follows:

In any relativistic equation containing a four-vector, it must always be possible to get the equation into a form in which each component of the four-vector is treated in the same way as every other component. The reason is that any preferential treatment of one or more of the components would imply an anisotropy of the equation in space-time, and thus there would exist rotations of the four-dimensional axes (i.e., Lorentz transformations, or rotations in three-space, or both) that would change the form of the equation; hence the equation would not be invariant to the Lorentz group.

Now if one undertakes to set up a linear relativistic Hamiltonian using only three different anticommuting matrices, the problem arises of finding a fourth matrix in order that each of the four components of the energy-momentum four-vector may have a matrix to go with it. One possibility is to use for the fourth matrix one of the three original matrices or a numerical multiple of it; then two of the components will be multiplied by commuting matrices and the others by anticommuting matrices, so that the components are not all treated alike. This is the situation in Mitler's Eq. (9). A second possibility is to use for the fourth matrix the unit matrix or a numerical multiple of it; but this commutes with everything so that we again have the difficulty of preferential treatment. Mitler's Eq. (11) uses the unit matrix and also uses one of the three original matrices in two different places.

It may be of interest to note that Pauli has given² a linear relativistic Hamiltonian using three two-rowed anticommuting matrices and using a numerical multiple of the unit matrix to go with the fourth component of the four-vector. This equation is intended to be used only for the case of a particle of rest-mass zero. Even with this restriction, however, Pauli's equation still fails to treat all the components of the energy-momentum four-vector alike, and is therefore not relativistically invariant.

The foregoing remarks should not be construed as deprecating Mr. Mitler's work. Dirac's equation is one of

the most complicated and unintelligible developments in all of theoretical physics, and in my opinion any attempt to find something simpler ought to be encouraged.

Michigan State College
East Lansing, Michigan

WILLIAM T. PAYNE

¹ Henri Mitler, *Am. J. Phys.* **21**, 473 (1953).

² Pauli *Handbuch der Physik* (Springer, Berlin, 1933), Vol. 24/1, p. 226.

Linearization of a Hamiltonian

IN reference to the letter titled "On the linearization of a relativistic hamiltonian," appearing in the September, 1953 issue; there is no paradox involved, but an error.

Numbered equations appearing in the previous letter shall be distinguished from those in this one by priming the former.

The error lies in that Eq. (2') is false. For the right-hand side is a matrix, and for Eq. (2') to be meaningful, the left-hand side must also be a matrix. Thus if A is a scalar, we must mean, by the left-hand side of Eq. (2') AI , where I is the 2×2 identity matrix. But two matrices are equal if, and only if, their corresponding elements are equal. Hence, denoting

$$\alpha_j = \begin{bmatrix} a_j & b_j \\ c_j & d_j \end{bmatrix}, \quad (1)$$

Eq. (2') becomes

$$\begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} = \begin{bmatrix} \sum a_j q_j & \sum b_j q_j \\ \sum c_j q_j & \sum d_j q_j \end{bmatrix}, \quad (2)$$

where the summation is over $j=1, 2, 3, 4$; and we must have, according to the statement appearing just above Eq. (1), (taking just the upper left-hand terms),

$$A = \sum a_j q_j, \quad (3)$$

a scalar equation for which we know there is no set of constant a_j 's satisfying it.

Hence we must mean by "linearizing," not

$$A = \sum \alpha_j q_j, \quad (2')$$

but

$$A^2 = (\sum \alpha_j q_j)^2. \quad (4)$$

Thus, we can no longer pass from Eq. (8') to Eq. (9'), on the assumption that Eq. (2') holds, but must pass directly from Eq. (7') to the square of Eq. (9'); that is, Eq. (7') implies

$$(\alpha_1 A + \alpha_2 i q_2)^2 = (\mathbf{a} \cdot \mathbf{q})^2, \quad (5)$$

and this is a matrix equation, so that we cannot simply take the square root of both sides, as a matrix has an infinite number of square roots. (For example,

$$J = \begin{bmatrix} 1 & x \\ 0 & -1 \end{bmatrix},$$

where x is any quantity, is a square root of I .)

But clearly Eq. (5) is *not* a linearization in the sense of Eq. (2'). Hence no paradox is involved, since we cannot pass from Eq. (5) to an equation of the form (4).

Palmer Physical Laboratory
Princeton University
Princeton, New Jersey

HENRI MITLER

On the Linearization of a Relativistic Hamiltonian

THE error in the letter of H. Mitler¹ arises from the assumption that the left-hand side of Eq. (8) can be linearized in the fashion indicated. In fact, since A is an operator which cannot be assumed to commute with α_1 or α_2 ,

$$(\alpha_1 A + i\alpha_2 q_4)^2 = \alpha_1 A \alpha_1 A + i\alpha_1 A \alpha_2 q_4 + i\alpha_2 \alpha_1 A q_4 - q_4^2 \neq A^2 - q_4^2.$$

University of Colorado
Boulder, Colorado

WESLEY E. BRITTIN

¹ Am. J. Phys. 21, 473 (1953).

On the Linearization of a Relativistic Hamiltonian

THE "apparent paradox" in a recent letter with the above title¹ appears to arise essentially from the confusion of a scalar with a matrix expression. In the following discussion, equations are numbered as in reference 1; the lettered equations are additional.

It was proposed¹ to linearize the expression

$$A = (q_\nu q_\nu)^{1/2}, \quad \nu = 1, 2, 3, 4 \quad (1)$$

using only the identity matrix I and the 2×2 Pauli matrices, $\alpha_1, \alpha_2, \alpha_3$, by writing

$$A^2 - q_4^2 = q_j q_j, \quad j = 1, 2, 3 \quad (7)$$

or, equivalently,

$$(A^2 + (iq_4)^2)^{1/2} = \pm (q_j q_j)^{1/2}. \quad (8)$$

However, the next equation

$$\alpha_1 A + \alpha_2 i q_4 = \pm \alpha_j q_j, \quad (9)$$

which leads directly to Eq. (11) of reference 1 and to the proposed linearization of A , does not follow. It is true that the expression $(A^2 + (iq_4)^2)^{1/2}$ can be linearized with the Pauli matrices and might be written

$$(A^2 + (iq_4)^2)^{1/2} = \alpha_1 A + \alpha_2 i q_4, \quad (a)$$

but this is not precise because the left member is scalar whereas the right member contains matrices. Frequently, no contradiction arises if one simply multiplies the left member by I , making a proper matrix equation. However, this is actually a subsidiary condition, and, in general, all that can be said is

$$\beta(A^2 + (iq_4)^2)^{1/2} = \alpha_1 A + \alpha_2 i q_4, \quad (b)$$

where β is a matrix whose square is I . Similarly for the linearization of $(q_j q_j)^{1/2}$,

$$\gamma(q_j q_j)^{1/2} = \alpha_j q_j, \quad \gamma^2 = 1, \quad (c)$$

not $(q_j q_j)^{1/2} = \alpha_j q_j$.

If one could put $\beta = \gamma = I$, Eq. (9) would follow; but Eq. (1) must hold in addition to Eq. (b) and Eq. (c). Let us try to determine γ so that all three equations are satisfied. There is little loss of generality in taking $\beta = I$. Multiplying Eq. (b) by β and Eq. (c) by γ and substituting into Eq. (8) gives

$$\alpha_1 A + \alpha_2 i q_4 = \pm (\gamma \alpha_j q_j).$$

Since $\alpha_1^2 = 1$, solving for A gives

$$A = \pm (\alpha_1 \gamma \alpha_j q_j) - \alpha_1 \alpha_2 i q_4 = \pm (\alpha_1 \gamma \alpha_j q_j) - \alpha_2 q_4, \quad (d)$$

which is all that follows from Eq. (b) and Eq. (c) and which is the correct expression to replace Eq. (11) of reference 1. [That Eq. (11) is incorrect can be verified directly by squaring the equation and comparing members.] Writing $\xi_1, \xi_2, \xi_3, \xi_4$, for the four matrices $\pm (\alpha_1 \gamma \alpha_j)$, $-\alpha_2$, it is easy to see that if Eq. (1) is satisfied, the ξ_μ must satisfy the usual Dirac conditions

$$\xi_\mu \xi_\nu + \xi_\nu \xi_\mu = 2\delta_{\mu\nu}. \quad (e)$$

Hence γ must be chosen to give, in suitable combination with the three α 's, four matrices which satisfy Eq. (e); and it is well known that four such 2×2 matrices do not exist. Thus γ can not be suitably chosen unless one goes to 4×4 matrices, as in the usual discussion of the Dirac matrices.

It was suggested in reference 1 that the step from Eq. (7) to Eq. (8) might be invalid. This step, however, is not at fault since all quantities in both equations are algebraic numbers. The misstep is in the incorrect transfer from the original scalar equations to suitable matrix equations.

E. J. ZIMMERMAN

University of Nebraska
Lincoln, Nebraska

¹ Henri Mitler, Am. J. Phys. 21, 473 (1953).

Note Concerning Radiation Pressure Against Perfect Reflectors

IN a recent paper¹ the following expression for radiation pressure was obtained:

$$p = u \left(1 - \frac{l}{c} \frac{\partial c}{\partial l} \right), \quad (1)$$

where u is the wave-energy density in front of the reflector, c is the phase velocity, and l is the distance from the reflector to another (possibly hypothetical) reflector that serves to establish a standing wave pattern between itself and the original reflector. The derivative $\partial c / \partial l$ is taken under the conditions demanded by the physical situation for an adiabatic virtual displacement of the reflector.

In an application of Eq. (1) l always drops out of the result, and is replaced by parameters of the medium propagating the waves.

A similar development is given in a paper by Post² that has just come to the author's attention. In Post's extensive bibliography he attributes the development to Brillouin.³ Post's paper also pointed up the following: It was concluded in the author's paper that Eq. (1) "indicates that for any situation where the phase velocity c is not altered by moving the reflector, the radiation pressure is just equal to the energy density." This conclusion is correct, but the further conclusion, "In physical situations where $p \neq u$, the virtual displacement must bring about some change in the medium that results in a change in c ," is subject to one important exception.

When the frequency of the wave motion lies in a region of dispersion, the virtual displacement brings about a change in c without "altering the medium." Equation (1) does properly account for this, though the above statement does not.

It is interesting to note that $(l/c)(\partial c/\partial l)$, "under conditions demanded by the physical situation," can be broken into two parts. One part $(l/c)(\partial c/\partial l)_d$ is due to dispersion; the other part $(l/c)(\partial c/\partial l)_f$ is due to the rest of the contributing factors, the "changes in the medium." The first

part satisfies the relations

$$\frac{l}{c} \left(\frac{\partial c}{\partial l} \right)_d = \frac{l}{c} \frac{\partial c}{\partial \lambda} \frac{\partial \lambda}{\partial l} = \frac{\lambda}{c} \frac{\partial c}{\partial \lambda}.$$

The group velocity g is $g = c - \lambda(\partial c/\partial \lambda) = c(1 - \lambda/c \partial c/\partial \lambda)$. Thus

$$p = u \left\{ 1 - \frac{\lambda}{c} \frac{\partial c}{\partial \lambda} - \frac{l}{c} \left(\frac{\partial c}{\partial l} \right)_f \right\} = u \left\{ \frac{g}{c} - \frac{l}{c} \left(\frac{\partial c}{\partial l} \right)_f \right\}.$$

Post² points out that the properties of these two contributions to pressure are different. The term ug/c is directed along the direction of wave propagation while the term involving a change in the properties of the medium by a virtual displacement of the reflector constitute a hydrostatic pressure.

It is regretted that these two prior papers were unknown to the author at the time of writing the above-mentioned paper.

J. E. RHODES, JR.

Georgia Institute of Technology
Atlanta, Georgia

¹ J. E. Rhodes, Jr., *Am. J. Phys.* **21**, 683 (1953).

² E. J. Post, *J. Acoust. Soc. Am.* **25**, 55 (1953).

³ L. Brillouin, *Rev. Acoust.* **5**, 99 (1936).

ANNOUNCEMENTS AND NEWS

Book Reviews

Introduction to Electron Microscopy. CECIL E. HALL.
Pp. 451+ix, Figs. 337, 15×24 cm. McGraw-Hill Book Company, Inc., New York, 1953. Price \$9.00.

This is it! Professor Hall's book is an answer to the need expressed at the Cleveland meeting of the Electron Microscope Society of America (November, 1952) for a college textbook in the physical principles and methods of electron microscopy, critically evaluated. In the selection and presentation of topics in electron optics, construction of electron microscopes, scattering phenomena, characteristics of electron images, techniques of specimen preparation, and examples from applied electron microscopy, it is evident that the author's primary purpose is that of imparting to the would-be electron microscopist such principles and methods of thought that he may be able to make progress on his own in analyzing and solving the unpredictable problems which await any investigator. It is this emphasis on systematic use of principles as the means for finding answers in general that seems best to justify inclusion of so specialized a field as electron microscopy in the formal program of a college student or graduate student.

Theory, including the elementary electron optics of lenses, aberration, diffraction, and interaction of electrons with matter, occupies about 45 percent of the text. Some 20 percent is devoted to the general construction of electron microscopes and a comparison of commercial models, and 20 percent to methods of calibration and specimen preparation. The remainder is a brief survey of applications of

electron microscopy, with many illustrations from biological work in which the author has participated.

Lest the space devoted to theory seem to some prospective microscopists disproportionate to their needs, one might add the author's own comment "... this book is intended for the practical electron microscopist and not for the electron optical theorist or the electron-microscope designer. If the discussion should seem too theoretical at times for the purpose, it may be stated that if all parts were deleted that were not a direct answer to questions asked at one time or another by practical persons engaged in laboratory work, very little shrinkage would occur."

The references given with each chapter appear to have been well selected from a prolific literature. There are 33 numerical problems. For class use, one could have desired more. Answers are given to 11.

The lucid style and broad horizons of *Electron Microscopy* lead a reader to feel that the author's own course must be a stimulating and valuable experience. The textbook is a distinguished addition to the publisher's *International Series in Pure and Applied Physics*.

ROBERT L. WEBER

The Pennsylvania State College

Introductory General Physics. JOHN G. WINANS. Pp. 765 +viii, Figs. 965, 15×23 cm. Ginn and Company, Boston, 1952. Price \$5.75.

The professor who considers using this textbook needs to consider whether or not two features of it will mesh with his present laboratory manual. These features in-

volve the choice of the three fundamental quantities and the choice of the direction of electric current. The three fundamental quantities adopted are length, force, and time. The electron flow is used for the direction of the current. The text assumes that in high school the student has had one year of algebra and one year of geometry. Chapter 2 is devoted to "The Algebra and Geometry of Physics." A knowledge of trigonometry is unnecessary; in fact the only trigonometric function defined is the sine, which is necessary for Snell's law. When components of a vector are desired in the solution of a problem, the components are obtained from a drawing made to scale.

The author has a preference for force, rather than mass, as a basic quantity since he feels that students have a general understanding of force and weight before beginning the study of physics. In the first chapter, entitled "Words and Measurements," the statement is made that "the gram force could be defined as one-thousandth part of the weight of a standard object known as the standard kilogram when that object is located in the Bureau of Standards at Washington, D. C." First use of the dyne is made on page 93, under surface tension. The author explains that the values of surface tension are so small that it is customary to employ a smaller unit of force, the dyne, rather than the gram. Seventy pages later the student is introduced to five systems of units: gravitational and absolute British and metric units and mks units. These various units are given in a table for twenty-one common physical quantities. Each quantity is designated as either vector or scalar. A similar table appears later for units in electricity and magnetism. Mass and weight are discussed in the chapter on Newton's laws of motion. In this chapter and in the following chapter discussing impulse and momentum, the symbol m rather than W/g appears in equations. The value of g is given as 32 ft/sec^2 or 980 cm/sec^2 . The only clues given to the student concerning variation of g are the statement mentioned above concerning the standard kilogram and a table hidden in the appendix where the metric values of g are listed for eight widely scattered localities.

The text applies Coulomb's law to magnetic poles and discusses the molecular theory of magnetism. The figure, long familiar in the old textbooks, showing the tiny molecular magnets lined up head-to-tail, appears in this book. As already mentioned, the electron flow is adopted as the current direction. Hence the left-hand rule is used to determine the direction of the magnetic field. In the chapter on static electricity and capacitance only electrostatic units are employed. The writer feels that illustrative examples could well be incorporated here to introduce the volt and the farad. After the student is introduced to electrostatic units, he is introduced to electromagnetic units and finally to mks units. In a later chapter on electric waves and radio practical units are employed.

At least one carefully drawn figure appears on almost every page. There are nearly a thousand figures in the book. All the usual simple lecture demonstrations are illustrated in pictures or in well-drawn figures. There are numerous illustrative examples, particularly in the first

part of the book. In these problems the data are listed and the principles or laws in equation form are written down. The numerical data, including units, are then substituted in the equations. Few answers are given for the numerous problems at the ends of the chapters. Although there are few illustrative examples in the part on light, there are many good ray diagrams and sketches. Care has been given to the make-up of the pages so that the book presents a pleasing and readable appearance to the student. There are discussions on the transistor, television, the jet plane, and fission. Only fusion seems to be missing.

If the textbook is adopted by a college which offers only one introductory course to which the students come with a variety of high school experiences, some well-prepared students may feel insulted by the mild use of mathematics. They may also have experienced mass as one of the fundamental quantities. Such students should not jump to the conclusion, however, that the text is "easy." It contains meat, is adequately complete, and the problems are good. In reading and studying the book, the average student will need little outside assistance. The conversion tables for units are excellent. In writing his textbook, the author has used as his guiding principle (as stated in the preface) that "teaching is lighting a candle, not filling a bushel."

G. K. SCHOEFFLE
Kent State University

A Laboratory Manual of Experiments in Physics. LEONARD ROSE INGERSOLL, MILES J. MARTIN, AND THEODORE A. ROUSE. Sixth Revision, Pp. 286, Exps. 80, Figs. 131, $16 \times 24 \text{ cm}$, McGraw-Hill Book Company, Inc., New York, 1953. Price \$4.00.

Comparison between Ingersoll's *Laboratory Notes in Physics* of 1923 and the sixth edition, with a third co-author (TAR), will illustrate well the marked evolution in laboratory resources and in instructional procedures during the past thirty years. Many principles and ideas now used in experiments were not known in the earlier days. Incident with changes in instructional procedures, the total pages per experiment have increased in the approximate ratio of two to four. Compared with the previous edition, the relationship between foreword, experimental procedure and supplemental material is basically the same.

The sixth edition initiates a series of more ambitious experiments. The seven new experiments require an average higher technical achievement level than those in former editions. Coupled with a deletion of thirteen experiments and with other rearrangements, the manual has a considerably changed and improved experimental approach for the better students. It is particularly useful in colleges and universities where better-than-average investment in apparatus per experiment is possible and where lecture, recitation, and laboratory are not only closely coordinated but taught by an experienced instructor. The book provides many examples of refined experimental techniques for the first course. High-precision timing experiments in mechanics and the better methods in dealing with heat losses in the experiments on heat are carried over with revisions from the former edition. Excellent use has been

made of the method of differences for treatment of data in several experiments. Helpful changes in directions were noted. The new write-up on magnetic fields has been considerably improved and clarified because of the extensive change in the description of Part III, coupled with a diagram. Many prospective users might welcome more changes in problems and in the numerical data of problems.

Though the idea back of five of the new experiments is laudable, principles of a more difficult nature are introduced, and the required equipment may be too expensive to purchase or maintain in the laboratory space available for those who wish to work with one experiment only per period. A few experiments carried over from the fifth edition are likewise too expensive for some laboratories operating on an even-front basis. Gradual simplification of design will eventually bring an increasing number of the newly introduced experiments into the average laboratory. They illustrate important physical principles and trends in measurements.

In institutions where instruction in the laboratory is left more to graduate students, there will be occasions when the mathematical treatment will be too brief and omissions of certain experimental details will be of concern to students at the lower end of the educational ladder in preparation and in instruction. More self-contained laboratory textbooks with closely integrated instruction is indicated for such students and is representative of an educational trend for those who hope to teach the average student a few important principles well. More pages per experiment will be required for additional figures, for more comprehensive instructional approach, and for accompanying mathematical techniques.

While many laboratory instructors have voiced vigorous complaints with respect to laboratory practices, including textbooks, present studies indicate possible directions in which progress lies. Many of the above remarks were made to emphasize the difficulties confronting authors of laboratory notes, manuals, and texts with respect to number and type of experiments, and associated theory. The sixth edition of *A Laboratory Manual of Experiments in Physics* should continue to be popular and widely used. It contains an excellent introductory section and many helpful directions on apparatus in Appendix 1, as well as many useful tables in Appendix 11.

LLOYD B. HAM
University of Arkansas

Practical Aids for Teachers of Physics

A large demonstration atomizer. Ordinary atomizers, used to dispense commercial products such as perfumes, are so small that they are not practical for use in demonstrating the principle of operation. The large demonstration atomizer illustrated in Fig. 1 is more satisfactory for this

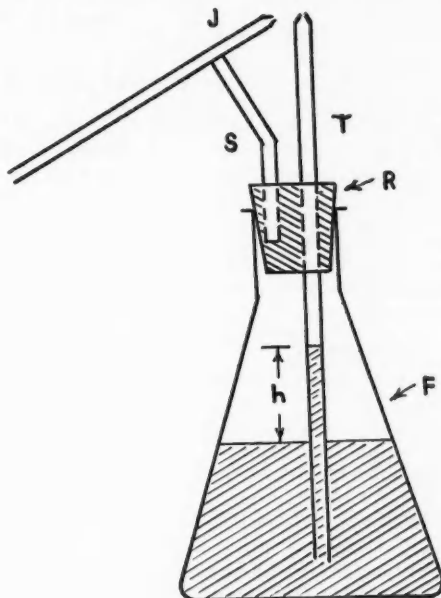


FIG. 1. Demonstration atomizer. Compressed air issues from jet *J* and produces a low-pressure region at the top of the 10-mm glass tube *T*. *F* is a 500-ml (or larger) Erlenmeyer flask. *S* is the support for *J*. The rubber stopper *R* must be vented to the atmosphere.

purpose. The liquid (colored water) rises in tube *T* as compressed air issues from jet *J*. *T* is large enough so that the level of the liquid is visible from all parts of a large lecture room. By controlling the velocity of the air coming from *J*, the level of the liquid in *T* can be made to stand at any desired height such as *h*. As the velocity is increased, the level rises; and when it reaches the top of *T*, the liquid can be sprayed out into the room for a distance of six feet or more. (Contributed by HARALD C. JENSEN, Lake Forest College, Lake Forest, Illinois.)

NEW MEMBERS OF THE ASSOCIATION

The following persons have been made members or junior members (*J*) of the American Association of Physics Teachers since the publication of the preceding list [*Am. J. Phys.* **21**, 704 (1953)].

- Anderson**, Frank Burr (*J*), Department of Physics, Williams College, Williamstown, Mass.
Bark, Maurice Joseph (*J*), 4489 Broadway, New York 33, N. Y.
Bartel, Peter S., North Newton, Kan.
Beddow, Frederic, 4957 Gateshead, Detroit 36, Mich.
Bents, Ulrich H., Department of Physics, Texas A and M, College Station, Texas
Beust, John Henry, St. Paul's School, Concord, N. H.
Beyer, Robert Thomas, 91 Leroy Drive, Riverside 15, R. I.
Box, E. O., East Central State College, Ada, Okla.
Cambron, Emile M., 1180 Bleury, Montreal 2, Quebec, Canada
Cember, Herman, 5458 Wilkins Ave., Pittsburgh, Pa.
Clarke, Robert A., 1624 24th St., Ogden, Utah
Climenhaga, John Leroy, Victoria College, Victoria, B. C., Canada
Corben, Herbert Charles, Carnegie Institute of Technology, Pittsburgh 13, Pa.
Cunningham, Robert Hanson (*J*), 1134 East First South, Salt Lake City, Utah
Davis, Morris S., Dept. of Physics, Univ. of North Carolina, Chapel Hill, N. C.
Dreyfuss, Maurice, 68 rue Villiers de l'Isle-Adam, Paris, France
Franklin, Allen Raymond (*J*), 430 E. Bloomington St., Iowa City, Iowa
Freier, George David, 2327 Gordon Ave., St. Paul 8, Minn.
Georgai's, René Louis, Lycée Claude Bernard, Paris 16^e, France
Green, Edward H., Brooklyn College, Department of Physics, Bedford Ave., Brooklyn, N. Y.
Green, Thomas A., Scott Laboratory, Middletown, Conn.
Greer, Earl Vincent, 700 North Donald, Bethany, Okla.
Guinier, Georges Ernest, 11 Rue de la Planche, Paris 7, France
Harber, Richard Aloysius (*J*), Department of Physics, University of Illinois, Urbana, Ill.
Harris, Frank Bower, Jr., 23 Grant St., Utica, N. Y.
Harrison, Don Edward, Jr., Department of Physics, University of Louisville, Louisville, Ky.
Holzberlein, Thomas Milton, The Principia College, Elsah, Ill.
Jackson, Howard L., 2560 Tyrone St., Flint 4, Mich.
Jewsbury, Wilbur Gordon, Colchester, Ill.
Josephs, Jess J., Department of Physics, Boston University, 725 Commonwealth Ave., Boston 15, Mass.
Kassner, James Lyle, Jr., 1620 2nd Ave., Tuscaloosa, Ala.
Kearney, Dora Evelyn, P.O. Box 432, Fayette, Iowa
Kelly, Frederick Miles, 741 Oak St., Winnipeg, Manitoba, Canada
Labonté, Father Gérard, 1855 Rachel est, Montreal 34, Canada
Leonard, Stanley Lee, Elsah, Ill.
Liefeld, Robert James (*J*), 811 Vine St., Kent, Ohio
Love, Winston C., Marietta College, Marietta, Ohio
Lubrano, Jack Andrew, 127 Blackamore Ave., Cranston 10, R. I.
Madden, Hannibal Hamlin, Jr. (*J*), Meacham St. Barracks No. 11-C, Williamstown, Mass.
Majka, Joe W. (*J*), 6522 Pacific Ave., Tacoma 4, Wash.
Mayo, Elizabeth Elliott, 2605 Delor Ave., Louisville, Ky.
McLaughlin, Neil Peter, 18th and Thompson Sts., Philadelphia 21, Pa.
McQuillin, Richard James (*J*), 514 West Main St., Puyallup, Wash.
Nankivell, John Elbert, Peach Hill Rd., Montville, N. J.
Osmond, Charles Anson, 2880 Van Buren St., Ogden, Utah
Palmer, Merrill C., Westminster College, New Wilmington, Pa.
Petz, Irene Adele, 814 Hennepin St., LaSalle, Ill.
Phillips, Mrs. Margaret Casto, 114 Conway Ave., Norfolk 5, Va.
Pierce, Edward Franklin, 8 Berry St., Danvers, Mass.
Shorthill, Richard W. (*J*), 465 Garfield Ave., Salt Lake City, Utah
Swihart, Ray Charles (*J*), 1104 Home Ave., North Manchester, Ind.
Tausner, Menasha Joseph (*J*), 2729 Wallace Ave., Bronx 67, N. Y.
Taylor, Julius Henry, Morgan State College, Baltimore 12, Md.
Thornley, Willard C., 22820 Sheridan Ave., Dearborn, Mich.
Toll, John Sampson, 2 Newland St., Chevy Chase, Md.
Van Tubergen, Robert Peter (*J*), Apt. 11-C, Latham St., Williamstown, Mass.
Weigel, George Allen, Box 67, Storrs, Conn.
Wilks, William Taylor, Box 50, State Teachers College, Troy, Ala.

THE AMERICAN INSTITUTE OF PHYSICS

INCORPORATED

57 East 55 Street New York 22, N.Y.

MEMBER SOCIETIES:

American Physical Society

KARL K. DARROW, *Secretary*, Columbia University, New York 27, New York.

Optical Society of America

ARTHUR C. HARDY, *Secretary*, Massachusetts Institute of Technology, Cambridge 39, Massachusetts.

Acoustical Society of America

WALLACE WATERFALL, *Secretary*, 57 East 55th Street, New York 22, New York.

Society of Rheology

W. R. WILLETS, *Secretary*, Titanium Pigment Corporation, 99 Hudson Street, New York 13, New York.

American Association of Physics Teachers

R. F. PATON, *Secretary*, University of Illinois, Urbana, Illinois.

AFFILIATED SOCIETIES:

American Crystallographic Association

American Society for Metals

Electron Microscope Society of America

Physical Society of Pittsburgh

Physics Club of Chicago

Physics Club of Philadelphia

Cleveland Physics Society

Physics Club of the Lehigh Valley

Sigma Pi Sigma

ASSOCIATES:

Over seventy-five leading industrial enterprises and other institutions which contribute to the support of the Institute.

JOURNALS:

The Physical Review

S. A. GOUDSMIT, *Editor*, Brookhaven National Laboratory, Upton, Long Island, New York.

Reviews of Modern Physics

J. W. BUCHTA, *Editor*, University of Minnesota, Minneapolis, Minnesota.

Journal of the Optical Society of America

WALLACE R. BRODE, *Editor*, National Bureau of Standards, Washington 25, D. C.

The Journal of the Acoustical Society of America

FLOYD A. FIRESTONE, *Editor*, 57 East 55th Street, New York 22, New York.

American Journal of Physics

THOMAS H. OSGOOD, *Editor*, Michigan State College, East Lansing, Michigan.

The Review of Scientific Instruments

GAYLORD P. HARNWELL, *Editor*, Randal Morgan Laboratory of Physics, University of Pennsylvania, Philadelphia, Pennsylvania.

The Journal of Chemical Physics

CLYDE A. HUTCHISON, JR., *Editor*, Institute for Nuclear Studies, University of Chicago, Chicago 37, Illinois.

Journal of Applied Physics

ELMER HUTCHISSON, *Editor*, Case Institute of Technology, Cleveland 6, Ohio.

Physics Today

ROBERT R. DAVIS, *Editor*, American Institute of Physics, 57 East 55 Street, New York 22, New York.

OFFICERS

GEORGE R. HARRISON, *Chairman*
Massachusetts Institute of Technology

GEORGE B. PEGRAM, *Treasurer*
Columbia University

GOVERNING BOARD

J. W. BUCHTA

KARL K. DARROW

G. J. DIENES

S. A. GOUDSMIT

GEORGE R. HARRISON

FREDERICK V. HUNT

DEANE B. JUDD

HUGH S. KNOWLES

C. C. LAURITSEN

WILLIAM F. MEGGERS

PHILIP M. MORSE

BRIAN O'BRIEN

GEORGE B. PEGRAM

I. I. RABI

DUANE ROLLER

FREDERICK SEITZ

WILLIAM SHOCKLEY

RICHARD M. SUTTON

J. H. VAN VLECK

MARK W. ZEMANSKY

Consultant on Printing

MELVIN LOOS

ADMINISTRATIVE STAFF

HENRY A. BARTON, *Director*

WALLACE WATERFALL,
Executive Secretary

THEODORE VORBURGER,
Advertising Manager

RUTH F. BRYANS,
Publication Manager

EDITH I. NEFFEL,
Circulation Manager

KATHRYN SETZE,
Assistant Treasurer

THE American Institute of Physics was founded in 1931 as a federation of leading societies in the field of Physics. It combines into one operating agency those functions on behalf of physics which can best be done by the Societies jointly. Its purpose is the advancement and diffusion of physics and its applications to human welfare. To this end it publishes for itself or the Societies the nine journals listed on this page; promotes unity and effectiveness of effort among all who are interested in physics; renders numerous direct services to physicists and to the public; and cooperates with government agencies, national associations, educational institutions, technical industries, and others in such manner as to realize the opportunities and fulfill the responsibilities of physics as an important and constructive human activity.

CONTENTS

AMERICAN JOURNAL OF PHYSICS

VOLUME 22, NUMBER 2

FEBRUARY, 1954

Physics of the Glassy State. I. Constitution and Structure	E. U. CONDON	43
Films Selected for College Physics Courses	ROBERT L. WEBER	54
Ionization in Air Maintained by a Uniform Plane Circular Distribution of α Radioactivity	R. W. NICHOLLS	59
Some Characteristics of Military Research and Development	DEAN E. WOOLDRIDGE	62
Laboratory Teaching in General Education Courses	CLEMENT L. HENSHAW	68
Reproductions of Prints, Drawings, and Paintings of Interest in the History of Physics. 58.		
<i>Retombera-t-il?</i>	E. C. WATSON	76
Wave and Inertial Properties of Matter	RICHARD SCHLEGEL	77
Quantum Mechanics and Thermodynamic Continuity. II.	ALFRED LANDÉ	82
Installation and Operation of Ten-Kilocurie Cobalt-60 Gamma-Radiation Source		
. . . JOHN V. NEHEMIAS, L. E. BROWNELL, W. W. MEINKE, AND E. W. COLEMAN		88
Notes and Discussion:		
Application of Matrix Algebra to Electric Networks	R. A. HOWARD	93
Equivalence of the Clausius and Kelvin Postulates	CHARLES N. HAMTIL	93
Concerning the Frequencies Resulting from Distortion	F. W. VAN NAME, JR.	94
Letters to the Editor:		
Linear Relativistic Hamiltonians	WILLIAM T. PAYNE	95
Linearization of a Hamiltonian	HENRI MITLER	95
On the Linearization of a Relativistic Hamiltonian	WESLEY E. BRITTIN	96
On the Linearization of a Relativistic Hamiltonian	E. J. ZIMMERMAN	96
Note Concerning Radiation Pressure Against Perfect Reflectors	J. E. RHODES, JR.	96
Announcements and News:		
Southern California Section		87
Central Pennsylvania Section		92
Book Reviews		97
Practical Aids for Teachers of Physics		75, 99
New Members of the Association		100